Keywords. Modern guidance law, Augmented proportional navigation, Zero-effort-miss

6.3 MODERN GUIDANCE LAWS

Essentially the modern guidance laws attempt to take into account target accelerations (i.e., target maneuvers), the time delay in the missile autopilot, and the effect of noise and uncertainty in measurement and estimation, as these are responsible for a missile's failure to perform well. Theoretically, many of them succeed in achieving their goals, but implementation of such laws become difficult due to their inherent complexity. Much of the application research on missile guidance is focused on the problem of efficient implementation of these laws or incorporation of some of their desirable features in classical guidance laws. Below we shall discuss two such guidance laws : Augmented Proportional Navigation (APN) and the Modern Guidance Scheme (MGS). Both can be considered to be extensions of the classical PN law in a linearized geometry. Indeed, we shall develop these guidance laws as logical extensions to the PN law, though their actual design could be done more rigorously through optimal control theory.

Augmented Proportional Navigation

In Fig.6.7 we show a missile-target geometry with a non-maneuvering target, and with a small angle assumption, that is, we assume the LOS angle θ to be small. This assumption allows the system to be treated as a linear one. Note that y is the vertical separation between the missile and the target. The time for intercept is t_f and so,

$$t_{go} = t_f - t$$

The commanded missile latax for the classical PN law is given by

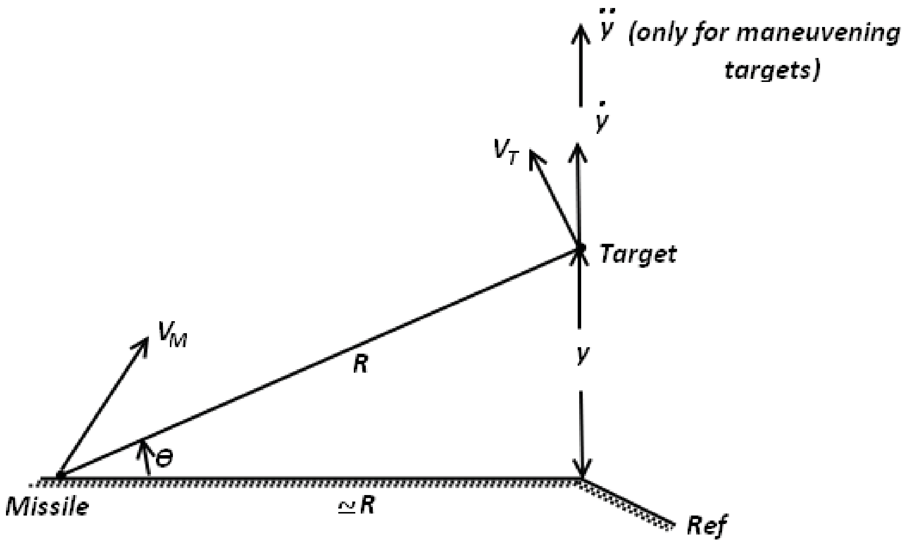

Figure 6.7: Small angle missile-target engagement geometry

$$a_M = NV_c\dot{\theta} = NV_c\frac{d\theta}{dt}$$

Note that t_{go} can be written as,

$$t_{go} = R/(-\dot{R}) = R/v_c$$

and so,

$$R = V_c t_{go}$$

Since the angle θ is small,

$$\theta \cong sin\theta = \tfrac{Y}{R} = \tfrac{Y}{V_c t_{go}}$$

Then we have

$$a_M = NV_c \frac{d}{dt}\left(\frac{y}{V_c t_{go}}\right) = N\frac{d}{dt}(yt_{go}^{-1}) \tag{6.1}$$

$$= N\{\dot{y}t_{go}^{-1} + y(-1)t_{go}^{-2}(-1)\} \tag{6.2}$$

$$= N\{\dot{y}t_{go}^{-1} + yt_{go}^{-2}\} \tag{6.3}$$

$$= \frac{N}{t_{go}^2}(\dot{y}t_{go} + y) \tag{6.4}$$

Now, let us analyze the expression within brackets in the above equation. It represents the (vertical) miss-distance that will occur at the end of time t_{go} (i.e., at final time t), provided that the missile does not apply any latax. Thus, this quantity is called the Zero-Effort-Miss (ZEM). Hence, the commanded missile latax for the classical PN law, in linearized geometry, can be written as,

$$a_M = \frac{N}{t_{go}^2}(ZEM) \tag{6.5}$$

So, proportional navigation (PN) in the small angle case turns out to be a guidance law which generates a latax command proportional to the ZEM and inversely proportional to the square of the time-to-go.

We shall now extend this idea further to obtain the APN guidance law. Suppose, in the same small angle case, we have a maneuvering target such that it causes a non-zero $\ddot{Y}$. Note that in the previous case (PN) we had assumed a non-maneuvering target and so we had only a non-zero $\dot{Y}$ which was caused by the target velocity. Then, from simple kinematics, the ZEM is given by,

$$ZEM = y + \dot{y}t_{go} + (1/2)\ddot{y}t_{go}^2$$

$$= y + \dot{y}t_{go} + (1/2)a_{Tn}t^2go$$

where, $\ddot{y}$ $=a_{Tn}$ = target acceleration normal to the LOS. Then, the commanded latax is given by,

$$a_M = \frac{N}{t_{go}^2}\{Y + \dot{Y}t_{go} + (1/2)a_{Tn}t_{go}^2\} \tag{6.6}$$

This is the <u>Augmented Proportional Navigation</u> (APN) guidance law. However, the above equation represents the guidance law in a linearized geometry. The guidance law in the general non-linear geometry is given by,

$$a_M = NV_c\dot{\theta} + (1/2)Na_{Tn} \tag{6.7}$$

This can be easily deduced from the above equation. Note that in addition to the LOS rate and the closing velocity, the APN guidance law also requires the target acceleration normal to the LOS for its implementation. In fact this requirement is its main drawback since measurement (or estimation) of target acceleration is prone to noise.

The advantage of the APN guidance law, as compared to PN, is that the commanded latax is initially high but falls as the missile approaches the maneuvering target. This is shown in Fig.6.8 below. Though the APN law takes into account the target acceleration, it does not account for the time

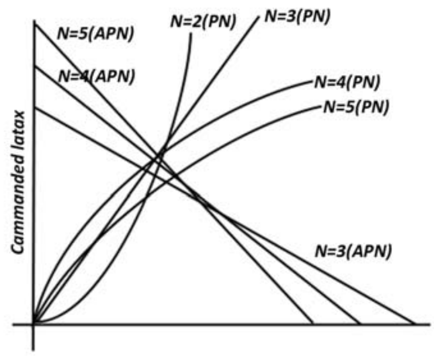

Figure 6.8: Command latax for PN and APN for different N and for maneuvering target

delay in the lateral autopilot of the missile which causes a difference between the commanded latax and the achieved latax. The modern guidance scheme described below takes this delay into account.

Modern Guidance Scheme

The modern guidance scheme is derived using the theory of optimal control. The latax that the missile pulls as it maneuvers induces a drag which affects its velocity. In an attempt to minimize this maneuver induced drag, the MGS guidance law is designed in such a way that it minimizes the following quantity (which is a measure of the maneuver induced drag) :

$$\int_0^{t_f} a_M^2 dt \tag{6.8}$$

under the condition that the terminal miss-distance is zero. In the linearized geometry (that is, small angle assumption) it means that $y(t_f)$=0. Solving this problem in a linearized setting, using optimal control theory, and assuming the autopilot to be a first order dynamical system, we obtain the MGS guidance law as,

$$a_{Mc} = \frac{N'}{{t_{go}}^2}\left\{y + yt_{go} + (\frac{1}{2})a_{Tn}t_{go}^2 - a_{Ma}\frac{(e^{-T} + T - 1)}{w^2}\right\} \tag{6.9}$$

where, a_{Mc} is the commanded latex and a_{Ma} is the currently achieved latex of the missile. Also,

$$T = wt_{go} \tag{6.10}$$

w=Bandwidth of the guidance system (autopilot) dynamics which is expressed as a first order time lag system

$$= \frac{1}{\tau}$$

where τ is the time constant in the first order system which models the lateral autopilot. Here, the navigation "constant" N is no longer a constant but is a time-varying quantity denoted by N'. It is given by,

$$N' = \frac{6T^2(e^{-T} - 1 + T)}{2T^3 + 3 + 6T - 6T^2 - 12Te^{-T} - 3e^{-2T}} \tag{6.11}$$

The expression within brackets in the expression for a_{Mc} above represents the ZEM. Note that the first part of the guidance law is identical to the APN law. In fact, as the time delay $t \to 0$, we have $T \to \infty$, we get a perfect autopilot. Also,

$$\lim_{T \to \infty} N' = 3 \tag{6.12}$$

Hence, for a perfect autopilot, the guidance command reduces to the APN guidance law with the optimal value of the navigation constant as 3. This justifies the choice of N as 3 to 5 in the PN and APN guidance laws. Translated to the non-linear settings the MGS law is expressed as,

$$a_{Mc} = N' V_c \dot{\theta} + N' \frac{1}{2} a_{Tn} - a_{Ma} \left\{ \frac{e^{-T} + T - 1}{T^2} \right\} \tag{6.13}$$

The MGS law has all the advantages of the APN guidance law in addition to the advantage that it takes care of the autopilot time delay. It suffers from the drawback that it requires the estimation or measurement of a_{Tn}, a_{Ma} and t_{go} which are difficult to measure accurately.

PROBLEMS AND EXERCISES

1. Consider the missile target engagement shown below. Answer the

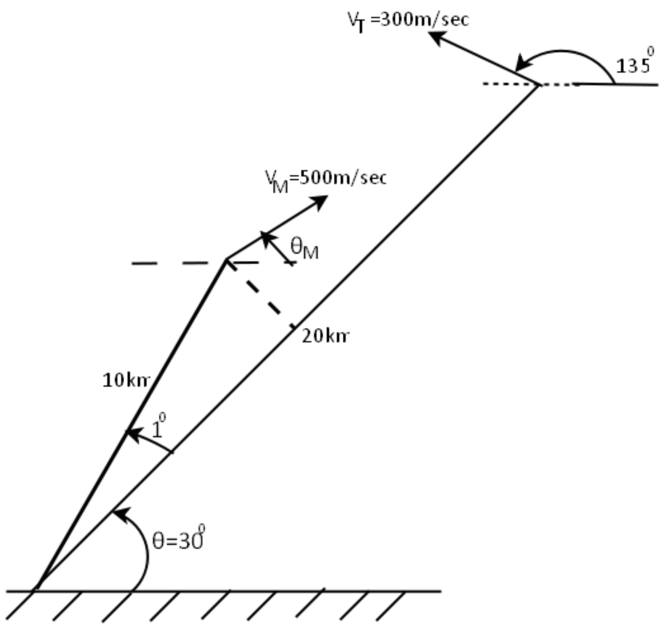

Figure 6.9:

following questions:

(I) what should be the angle θ_M in order that the condition for LOS guidance is met?

(II) Suppose the missile is guided by the pure pursuit guidance law then what should be θ_M in order to satisfy the condition of velocity pursuit and of attitude pursuit? The angle of attack is 2 degrees and the velocity vector lags the missile longitudinal axis.

(III) Answer (II) when the missile is guided by a deviated pursuit guidance law with the angel of deviation=2.5 degrees.

2. Consider the missile-target engagement geometry given below at some instant in time t. Answer the following questions:

(I) What is the commanded latex if the missile uses (a) PN with N=4 (b) APN with N=3 (c) MGS with the current achieved latex same as the commanded latex in (b) and the time delay t=0.1 sec?

(II) What is the estimated time-to-go? Is the actual time-to-go for the three guidance laws less, more or the same as the estimated value? Assume that the target employs a constant measure level, i.e., constant a_T throughout the engagement.

3. Obtain the value of N' when $T \to \infty$.

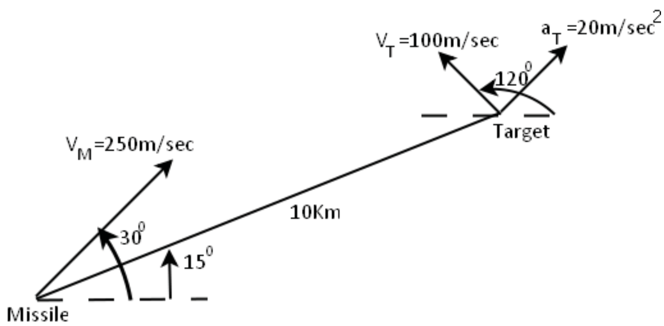
V_T =100m/sec
a_T =20m/sec^2
120^0
Target
V_M=250m/sec
10Km
30^0
15^0
Missile

Figure 6.10: