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Helicopter Wake Form Visualization Results and their Application to Coaxial Rotor Analysis at Hover




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Notation

A	minimum nondimensional wake radius
T, N	rotor thrust & power, kg, hp
C_T, m_K	rotor thrust & torque coefficients, $\frac{16 \cdot T}{\Delta \cdot F \cdot (\omega R)^2}$, $\frac{1200 \cdot N}{\Delta \cdot F \cdot (\omega R)^3}$
FM	figure of merit, $\frac{C_T \sqrt{C_T}}{2 \cdot m_K}$
R	rotor radius, m
F	rotor disk area, πR^2, m^2
K1, K2	axial slope of tip vortex trajectory before and after passage of the following blade
$\bar{y}$	relative axial coordinate, y/R
$\bar{r}$	relative radial coordinate, r/R
t	time, sec
$\bar{t}$	thrust ratio between upper & lower coaxial rotors, T_U / T_L
$\bar{\upsilon}$	relative velocity, $\upsilon/(\omega R)$
ψ	wake azimuth angle, deg.
λ	wake contraction rate parameter
ω	rotor angular speed, rad/sec

Subscripts

U, L	upper, lower
UR, LR	upper rotor, lower rotor
S, C	single, coaxial
AV	average
MAI	Moscow Aviation Institute, Russia
TsAGI	Central Aerohydrodynamics Institute named after N.E. Joukovsky, Russia

Introduction

Currently in calculations of rotor on axial flight modes the mathematical models on the basis which one are applied the vortical system both with freely deformed (soft), and with a rigid vortex wake of the preset form is stated. The mathematical models with a rigid vortex wake much easier and also allow to conduct calculation of the characteristics of rotor with adequate accuracy and as against models with a soft vortex wake demand considerably smaller machine resources.

In all cases an indispensable condition of development of numerical model of a vortex wake is the availability of experimental data about the form and velocities in wake of a rotor.

The present article contains the main results of visualization of trailing vortexes of the single-rotor Mi-4 helicopter and coaxial Ka-32 helicopter, obtained for at hover mode, in full-scale flight tests at Flight Research Institute (FRI) named after M.M. Gromov.

The results of joint researches of KAMOV company and FRI of the coaxial rotor wake form are published earlier in papers [1,6]. Level flight conditions and to a lesser degree hovering mode were researched. The activities on research of a hovering mode of the Ka-32 helicopter were proceeded. The results of these researches are submitted in the present article.

The known formulas of approximating of an outer boundary of a vortex wake for a single rotor are analyzed. The original formula of approximating for a coaxial rotor is offered. Comparative estimation of accuracy of approximating of trailing vortexes coordinates for Mi-4 and Ka-32 helicopters is performed.

With usage of the best approximating of the wake form, the calculations of aerodynamic properties of a coaxial rotor in a hovering mode are made. Comparison of calculations and flight tests results is submitted.

1. Flight researches and approximating of trailing vortex coordinates

1.1 Flight researches

As objects of research the helicopters - single-rotor Mi-4 and coaxial Ka-32 are utilized (Table_2). The researches were conducted on the basis of FRI [1,5,6].

All flight researches were conducted in hovering modes at OGI small altitudes. The rotors of the coaxial helicopter were balanced (torque moments brought to upper and lower rotors are equal).

The blades of helicopters were equipped with a system of smoke visualization. As generators of a smoke the special small-sized flares TD-26 of white and orange colour, designed PH research institute, Zagorsk, were used [5].

The unit consisting 4 smoke generators was established on the end of each blade under a tip fairing instead of navigation lights, that eliminated violation of an external contour of a blade profile. The valid time of the generator was about 10 sec. All flares on blades of one rotor were actuated simultaneously (on the Ka-32 helicopter generators of the second rotor were fired after termination combustion on the first). The smoke was exhausted through a slot of 45×6 mm arranged on 1/4 of blade chords length. That was earlier stated, that such extension of a smoke track visualizes a trailing vortex of a blade [1].

Filming a vortex wake was conducted by the fast-track camera with frequency about 100 frames per second. The filming helicopter was placed sideways or in front from object of filming on distance of 150...250 m.

The cross section of an outer boundary of a wake was obtained by V.P. Butov by processing of shots of frontal filming. Series on time the frames were projected on a screen, on which one the series positions of trailing vortexes were marked. To a zero reference datum on time there corresponded a position of trailing vortexes in a plane of blade tips. The error of an evaluation of an appropriate temporal step did not exceed 2 %.

The representative results of processing of shots for Mi-4 and Ka-32 helicopters, is shown in a Fig.3,6 accordingly.

1.2 Methods of approximating of trailing vortex coordinates

The known methods of approximating of vertical and radial coordinates of a trailing vortex can be divided into two groups:

I. Vertical coordinate is approximated as a function of azimuthal position of a vortex and design data of a rotor, and radial - as a vertical coordinate function:

$$\begin{cases} \bar{y} = f(\psi, c_T); \\ \bar{r} = f(\bar{y}). \end{cases}$$

Approximating of the first group realised in analytical expressions by C. Koning [8], E.D. Safronov [13], V.S. Vozhdaev [7].

II. The coordinates of a vortex core are approximated as a function of an azimuthal position of a vortex and design data of a rotor:

$$\begin{cases} \bar{y} = f(\psi, c_T, \Delta\varphi, \ldots); \\ \bar{r} = f(\psi, c_T, \Delta\varphi, \ldots). \end{cases}$$

Approximating the second group A.J. Landgrebe [10] has offered to use. It grounded on research of smoke visualization of a vortex wake of rotors models. Similar introducing have the theories of J.D. Kocurek and J.L. Tangler [9], V.E. Baskin [2], V.I. Shaidakov [15]. Such type of approximating obviously links parameters of a rotor and position of a trailing vortex, and also represents coordinates of a vortex as a function of time $\psi = \omega t$. The general format of introducing of vortex coordinates in this group looks like:

$$\bar{y} = \begin{cases} K1 \cdot \psi, & \text{for} \quad 0 \le \psi \le \dfrac{2\pi}{k_Л}; \\ K1 \cdot \dfrac{2\pi}{k_Л} + K2 \cdot \left(\psi - \dfrac{2\pi}{k_Л}\right), & \text{for} \quad \dfrac{2\pi}{k_Л} < \psi \le 4 \cdot \dfrac{2\pi}{k_Л}. \end{cases}$$

$$\bar{r} = A + (1 - A) \cdot EXP(-\lambda\psi), \quad \text{for} \quad 0 \le \psi \le 4 \cdot \frac{2\pi}{k_Л}.$$

As it is visible, from the listed writers of the theories of approximating of rotor vortex wake, the Russian school of aerodynamics is widely submitted. As against the Russian researches, the foreign methods of approximating grounded basically on study of experiments on rotor models.

The listed methods of approximating of trailing vortex coordinates for a hovering mode are shown in Table_1. For methods of the second group the formulas of calculation of factors K1, K2, λ, A are provided.

In line No.7 of Table_1 proposed by us original formula of approximating of trailing vortex radial coordinate is adduced. The formula can be utilised both for single, and for coaxial rotors. The formula is obtained by results of the analysis of visualization of trailing vortexes of the single-rotor Mi-4 helicopter and coaxial Ka-32 helicopter in a hovering mode in full-scale flight tests.

In a proposed method of calculation of radiuses of throat section of a wake of a coaxial rotor (A_U, A_L) - are counted on the basis of a simple physical analog of the momentum theory of a coaxial and equivalent single rotor at given of experiment to value (A) for a single rotor and usage of experimental relation $\bar{t} = f(\bar{h})$ (Fig.1,2). It is necessary to point out, that results of calculation of values (A_U, A_L) under the momentum theory will well be agreed with flight tests measurements (Fig.2).

1.3 Analysis of approximating methods

The results of polynomial approximating of coordinates of trailing vortexes obtained in flight tests for Mi-4 and Ka-32 helicopters are shown in a Fig.4, 7 accordingly.

For the Mi-4 helicopter radius of throat section(cross-section) of a wake A ≈ 0.86 (Fig.4) is obtained, instead of A ≈ 0.78 - obtained by A.J. Landgrebe [10], J.D. Kocurek and J.L. Tangler [9] at research of a pathway of trailing vortexes of single rotor models by means of smoke visualization and schlieren photography method.

For the Ka-32 helicopter it is necessary apart to mark availability so-called peculiarity feature of a jet edge on radial coordinate at lateral angles of a vortex ψ = 0...90° (Fig.7). The standard radius of radial contraction of a jet edge of the upper rotor in a plane of the lower rotor has compounded $\bar{r} \approx 0.85$ (Fig.7) for $\bar{h} \approx 0.2$. Relative vertical velocities K1, K2 of drift of trailing vortexes of upper and lower rotor differ unsignificantly (Fig.7). It allows to use for approximating vertical coordinate of trailing vortexes of a coaxial rotor of the formula for a single rotor. The similar results were obtained earlier at processing and analysis of flight tests of the Ka-32 helicopter [1].

The comparative results of approximating of trailing vortexes coordinates $\bar{y}$ and $\bar{r}$ for Mi-4 and Ka-32 helicopters formulas shown in Table_1, are shown in Table_3 and submitted in a Fig.5,8 accordingly. The input data are taken from the Table_2.

The analysis has shown, that J.D. Kocurek and J.L. Tangler [9] method (line No.5, Table_1) is the best for approximating vertical coordinate $\bar{y}$ of a trailing vortex as for single-rotor, and coaxial rotor too (Fig.5, 8).

Earlier, in paper [1] was stated, that the formulas of approximating of radial coordinate of a trailing vortex $\bar{r}$ of a single rotor can not be directly applied for a coaxial rotor. A V.S. Vozhdaev method [7] of calculation of trailing vortexes radial coordinates of a coaxial rotor $\bar{r}$ (line No.2, Table_1) gives a considerable error of approximating (Fig. 8).

The best result (Fig. 8) gives the suggested in the present article approximation (line No.7, Table_1), recorded as for A.J. Landgrebe by the way of exponential functions combination EXP(X) and EXP(-X), namely:

$$\bar{r} = A + \frac{(1-A)}{\operatorname{ch}(n\lambda\psi)} = A + (1-A)\cdot\frac{2}{\operatorname{EXP}(n\lambda\psi) + \operatorname{EXP}(-n\lambda\psi)},$$

where the analysis of experiment gives (Fig.1, 2):

$$\bar{t} = 2 - \operatorname{EXP}(-1.307\cdot\bar{h});$$

$$A_U = A\sqrt{(\bar{t}+1)/(2\cdot\bar{t})},\ A = 0.86;$$

$$A_L = A_U\sqrt{\bar{t}};$$

$$n_L = 4; \quad n_U = 2 + 2\cdot\operatorname{EXP}(-20\cdot\bar{h}).$$

Thus the value λ is calculated on a J.D. Kocurek and J.L. Tangler method [9] (line No.5, Table_1).

Having substituted $\psi = \omega t$ in the formula for $\bar{r}$ calculation, we shall discover radial velocity of contraction of a jet edge for suggested approximation:

$$\bar{\upsilon}_r = \frac{d\bar{r}}{dt} = -\frac{1-A}{\operatorname{ch}^2(n\lambda\omega t)}\cdot\operatorname{sh}(n\lambda\omega t)\cdot(n\lambda\omega) = -\frac{n\lambda\omega(1-A)}{\operatorname{ch}(n\lambda\omega t)}\cdot\operatorname{th}(n\lambda\omega t) = (A-1)\cdot n\lambda\omega\cdot\operatorname{th}(n\lambda\omega t)\sqrt{1-\operatorname{th}^2(n\lambda\omega t)}.$$

The suggested formula of $\bar{r}$ calculation at ($\bar{h}=0, \bar{t}=1, n=4, A=0.86$) with adequate accuracy approximates radial coordinate of a single rotor trailing vortex (Fig. 5).

2. Aerodynamic calculation of a coaxial rotor on a hovering mode

2.1 Premise

In Russia the development of the numerical methods of calculation of the coaxial rotor characteristics permanently progressed and has achieved such stage, on which one characteristic can be calculated with a high scale of accuracy, up to limits matched with accuracy of experiment.

The full review of the Russian idealized and experimental activities of a TsAGI, MAI and KAMOV company on aerodynamics of coaxial rotor in hovering modes is published earlier in article [3].

In the KAMOV company a lot of attention is given to development of the numerical methods of calculation of rotor performances. The calculation of the characteristics of a rotor on hovering developed from a simple momentum theory to simulation of a wake in process of increase of computers response. Proceedings of L.A. Potashnik, E.A. Petrosian, V.N. Kvokov, V.A. Anikin, B.N. Bourtsev - become the basis of Russian aerodynamics of coaxial rotors [3, 4].

2.2 Program of calculation of the coaxial rotor characteristics

In the KAMOV company the general mathematical model ULISS-6 of a coaxial carrier system was built for the solution of general problems of aeroelasticity [4]. The given program integrates:

- mathematical model of coaxial rotor with flexible blades;
- model of an elastic control system;
- vortical model of coaxial rotor.

The non-linear vortical model was developed on the basis of a lifting-plane theory. An idiosyncrasy is the solution of a system of linear algebraic equations determining condition nonpassing through of deforming lifting surfaces (blades) solved together with an integration of motion equations of flexible blades. Thus in each instant the conformity of deformations, air loads both intensities of lifting and free vortex surfaces is reached.

The vortical model of the ULISS-6 code allows to model motion of flexible blades of coaxial rotor at manoeuvres of the helicopter and on steady conditions of flight. Thus are calculated:

- loads on bushes of upper and lower coaxial rotor;
- general and spread loads on blades;
- loads in a control system;
- geometrical parameters of blades motion.

The modelled aeroelastic phenomena and functional capabilities of the ULISS-6 code are explained earlier in article [4, Fig.6].

2.3 Geometry of wake model of a rotor

The non-linear form of a wake in the ULISS-6 code depends on a flight phase and is approximated on the basis of results of trailing vortexes visualization of the Ka-32 helicopter [1].

In the present article the version of the ULISS-6 code for definition of rotor performances on hovering is considered. The accuracy of construction of geometrical model of a wake - is a key to successful application of the program in a broad band of computational parameters of a rotor. In a Fig. 9 the used vortical model of a coaxial rotor in hovering mode is schematically shown.

In wake model used by us of trailing vortexes velocity of upper and lower coaxial rotor are calculated (Fig. 10):

- vertical K1, K2 - on J.D. Kocurek and J.L. Tangler [9] method (line No.5, Table_1). Thus, during calculation, the value of blade linear twist $\Delta\varphi_\Sigma$ in the Kocurek and Tangler formulas is corrected with allowance for of elastic twist along the blade lenght. Then the form of a wake etc. by iterations is updated;
- radial $\overline{\upsilon}_r = d\bar{r}/dt$ - are calculated by method offered us, (line No.7, Table_1).

Velocities of free vortexes of an internal vortex wake (Fig. 10).

For the upper rotor in areas:

- (BEJKFC) - are peer K1, K2, $\overline{\upsilon}_r$ velocities of a trailing vortex of the upper rotor. Thus distance (BC) is received equal 1/4 chords of a blade;
- (ADIJEB) - vertical and radial velocity on the dextral boundary of area are peer (3/2)·K1, (3/2)·K2, $\overline{\upsilon}_r$ velocities of a trailing vortex of the upper rotor. Inside area they linearly changes up to 0 on an axis of rotor.

For the lower rotor in areas:

- (GLMH) - are peer K1, K2, $\overline{\upsilon}_r$ velocities of a trailing vortex of the lower rotor. Thus distance (GH) is received equal 1/4 chords of a blade;
- (FKLG) - the vertical velocity on the dextral boundary of area is peer (3/2)·K1, (3/2)·K2 of a velocity of a trailing vortex of the lower rotor. Inside area it linearly changes up to 0 on an axis of rotor. The radial velocity $\overline{\upsilon}_r$ on the dextral boundary of area is peer to a velocity of a trailing vortex of the lower rotor. The radial velocity on the left boundary of area is peer to a velocity of a trailing vortex of the upper rotor. Inside area of radial velocities are linearly interpolated;
- (DIJE) - vertical and radial velocity of vortexes coincide with velocities of vortexes from blades of the upper rotor and are located in the same space.

The free vortex surface is disjointed on "short-range", including longitudinal and transversal vortexes, and "distant", including only trailing vortexes, that models a phenomenon of swerving of a free vortex surface (Fig. 11).

The vortical system of coaxial rotors is represented by combination of vortical systems of blades.

2.4 Computation methods comparison

The described above version of the ULISS-6 code allows to calculate the characteristics of rotors both usual geometry and perspective rotor with the composite shape of a blade tip section. The example of calculation of such rotor of the Ka-32 helicopter is shown in Fig.13. The conducted evaluations well agrees with the data of flight tests.

In Fig.12,13 the evaluations under the ULISS-6 code and under the E.A. Petrosian momentum theory of a coaxial rotor [12] are shown.

In Fig.12 the distribution of magnetic velocities, angles of attack and lift coefficients on a blade of upper and lower coaxial rotor are presented. The distribution calculated by us on the ULISS-6 code in an internal part of blade ($\bar{r} \le 0.85$), on nature is similar to distribution obtained on the momentum theory [12]. On an external part of blade ($0.85 < \bar{r} \le 1.0$) the ULISS-6 code gives more precise loads distribution, effected by influencing of trailing vortexes.

Conclusions

1. All published approximating formulas of an outer boundary of a vortex rotor wake in hovering are pioneered in investigation reviewed. The original formula of approximating of radial coordinate both for single and coaxial rotors is first obtained.

The formulas of minimum wake radius of a coaxial and equivalent single rotors are obtained on the experimental data and based on momentum theory.

2. The published approximating formulas of an outer boundary of rotors vortex wake are compared with the data which have been obtained in full-scale flight tests of Mi-4 and Ka-32 helicopters.

3. The Kocurek's and Tangler's formulas are considered as optimal for approximating vertical coordinates both for single Mi-4 and coaxial Ka-32 helicopters rotors.

The present paper is authors obtained adequate approximating the formula of wake radial coordinates both coaxial and single rotors.

4. The version of the ULISS-6 code approximation geometry of a wake in a hovering mode is effectively applied for coaxial rotor performance data calculation.

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Table 1.

Wake Boundary Approximation Methods of the Main Rotor				
No.	Author	Method	Formula	Notes
1	2	3	4	5
1	C. Koning [8], 1940 E.D. Safronov [13], 1972	Rotor disk model	$\bar{y} = K \cdot \psi$; where: $K = \bar{\upsilon}_1 = \frac{1}{2}\sqrt{c_T}$. $\bar{r} = \sqrt{\dfrac{1}{1 + \dfrac{\bar{y}}{\sqrt{1+\bar{y}^2}}}}$.	Single Rotor
2	V.S. Vozhdaev [7], 1977	Simplified rotor blade model	$\bar{y} = K \cdot \psi$; where: $K = \bar{\upsilon}_1 = \frac{1}{2}\sqrt{c_T}$. $\bar{r}_U = \sqrt{\dfrac{1-\bar{h}/2}{1 + \dfrac{\bar{y}}{\sqrt{1+\bar{y}^2}} - \dfrac{1}{2}\cdot\dfrac{\bar{h}}{\left(1+\bar{y}^2\right)^{3/2}}}}$; $\bar{r}_L = \sqrt{\dfrac{1+\bar{h}/2}{1 + \dfrac{\bar{y}}{\sqrt{1+\bar{y}^2}} + \dfrac{1}{2}\cdot\dfrac{\bar{h}}{\left(1+\bar{y}^2\right)^{3/2}}}}$.	Coaxial Rotor
3	V.E. Baskin V.S. Kaplan [2], 1979	Sink method	$\bar{y} = -\dfrac{32}{9\pi}\left(\sqrt{1-\bar{r}^2} - \dfrac{\sqrt{5}}{6}LN\dfrac{\sqrt{5}+3\sqrt{1-\bar{r}^2}}{\sqrt{5}-3\sqrt{1-\bar{r}^2}}\right)$; $\bar{r} = \dfrac{2}{\sqrt{9-5\cdot EXP\left(-a\psi\sqrt{c_T}\right)}}$; $a = \pi\dfrac{\sqrt{3}}{4}$.	Single Rotor
4	A.J. Landgrebe [10], 1972	Smoke wake investigation on rotor models	$K1 = 0.25\cdot\left(t + 0.001\cdot\Delta\varphi_\Sigma\right)$; $K2 = t + 0.01\cdot\Delta\varphi_\Sigma\sqrt{c_T/2}$; where: $t = c_T/2\sigma$. $A = 0.78$; $\lambda = 0.145 + 13.5\cdot c_T$.	Single Rotor
5	J.D. Kocurek J.D. Tangler [9], 1976	Schlieren photography wake investigation on rotor models	$K1 = -\left[B + C\cdot\left(\dfrac{c_T}{2k_Л^N}\right)^M\right]$; $K2 = \sqrt{c_T/2 - c_{T0}}$; $A = 0.78$; $\lambda = 4\sqrt{c_T/2}$; where: $B = -0.000729\cdot\Delta\varphi_\Sigma$; $C = -2.3 + 0.206\cdot\Delta\varphi_\Sigma$; $M = 1 - 0.25\cdot EXP\left(0.04\cdot\Delta\varphi_\Sigma\right)$; $N = 0.5 - 0.0172\cdot\Delta\varphi_\Sigma$; $c_{T0} = k_Л^N\cdot\left(-B/C\right)^{\frac{1}{M}}$.	Single Rotor

Table 1 (Cont.)

1	2	3	4	5
6	V.I. Shaidakov [15], 1994	Simplified vortex wake model	$K1 = \tilde{\upsilon}_{1cor}\sqrt{c_T}$; $K2 = \tilde{\upsilon}_{2cor}\sqrt{c_T}$; $\lambda = 2\sqrt{2c_T}$; $A = \sqrt{\frac{\tilde{\upsilon}_{1AV}}{\tilde{\upsilon}_{2AV}}}$; where: $\tilde{\upsilon}_{1cor} = \frac{1}{2}\frac{\bar{h}_{cor}}{\pi}\left(A_1 + \frac{1}{2}B_1\right)$; $\tilde{\upsilon}_{2cor} = \frac{1}{2}\cdot\frac{\bar{h}_{cor}}{\pi}(A_1 + B_1)$; $\bar{h}_{cor} = \frac{2\pi}{k_{\text{Л}}}\cdot\frac{1}{2}\sqrt{c_T}$; $A_1 = 2.26 - \frac{1}{2}LN\left[1 - EXP\left(-1.25\cdot\bar{h}_{cor}\right)\right]$; $B_1 = \sum_{n=1}^{\infty} k_n\left[K(k_n) - E(k_n)\right]$, where: K, E - elliptic integrals, $k_n = \frac{2}{\sqrt{4 + (n\bar{h}_{cor})^2}}$; $\tilde{\upsilon}_{1AV} = C_1 + D_1$; $\tilde{\upsilon}_{2AV} = C_1 + 2D_1$; $C_1 = \frac{\bar{h}_{cor}}{\pi}\left(LN\left(\frac{8}{\bar{r}_{cor}}\right) - 2\right)$; $D_1 = \sum_{n=1}^{\infty}\frac{\bar{h}_{cor}}{2\pi}k_n\left[\left(2 + (n\bar{h}_{cor})^2\right)\cdot K(k_n) -\right.$ $\left. - \left(4 + (n\bar{h}_{cor})^2\right)\cdot E(k_n)\right]$; $\bar{r}_{cor} = 0.034\cdot\left[1 - EXP\left(-1.25\cdot\bar{h}_{cor}\right)\right]$.	Single Rotor
7	Proposed method	Full scale smoke wake investigation	y/R & λ values are calculated as in Kocurek & Tangler method (ref. to p.5). $\bar{r} = A + \frac{(1-A)}{ch(n\lambda\psi)} =$ $= A + (1-A)\cdot\frac{2}{EXP(n\lambda\psi) + EXP(-n\lambda\psi)}$; For Single Rotor ($\bar{h} = 0$, $\bar{t} = 1$): $A = 0.86$; $n = 4$. For Coaxial Rotor ($0 \le \bar{h} \le 0.4$) : $\bar{t} = 2 - EXP(-1.307\cdot\bar{h})$; $A_U = A\sqrt{(\bar{t}+1)/(2\cdot\bar{t})}$; $A = 0.86$; $A_L = A_U\sqrt{\bar{t}}$; $n_L = 4$; $n_U = 2 + 2\cdot EXP(-20\cdot\bar{h})$. For Ka-32 Helicopter ($\bar{h} = 0.2$) : $\bar{t} = 1.23$; $A_U = 0.819$; $A_L = 0.908$; $n_U = 2$.	Single Rotor & Coaxial Rotor

.

Table 2.

Basic Test Data & Main Rotor Data			The Russian Helicopters, Main Rotor, Single	The Russian Helicopters, Main Rotor, Coaxial
Parameter	Notation	Dimension	Mi - 4	Ka - 32
T-O weight	G	kg	6500	10000
Rotor load	P = G / F	kg / m^2	18.77	50.36
Number of blades	$k_Л$	-	4	3 + 3
Rotor radius	R	m	10.5	7.95
Upper / lower rotor hub distance	$\bar{h}_0 = h_0 / R$	-	-	0.189
Upper / lower rotor blade tip clearance *	$\bar{h} = h / R$	-	-	~ 0.2
Rotor blade chord	b	m	0.52	0.48
Rotor solidity	σ	-	0.063	0.1153
Blade linear twist	$\Delta\varphi_\Sigma$	deg	- 5	- 6
Blade taper	-	-	1 : 1	1 : 1
NACA airfoil	-	-	230M	230
Rotor tip speed	ωR	m / sec	197	226
Rotor thrust coefficient	C_T	-	0.0077	0.0158
Thrust coefficient solidity ratio	C_T / σ	-	0.1222	0.1370
Average induced velocity at hover	$\bar{\nu}_1 = \frac{1}{2}\sqrt{C_T}$	-	0.0439	0.0628
Relative air density	Δ	-	~ 1.0	~ 1.0

* $\bar{h} = \bar{h}_0 + \left(\mathrm{Sin}(a_{0\,U}) - \mathrm{Sin}(a_{0\,L})\right)$, where: $a_{0\,U/L} = \gamma_{U/L} \cdot \frac{C_{T\,U/L}}{8 \cdot \sigma_{U/L}}$; $\gamma = \frac{b \cdot \Delta \cdot a_\infty \cdot R^4}{16 \cdot I_{FH}\,(kg \cdot m \cdot sec^2)}$.

For Ka-32 Helicopter : $\gamma_{U/L} = 4.95$, $\sigma_{U/L} = \sigma / 2 = 0.05765$; $C_{T\,U} = C_T \cdot \bar{t} / (1 + \bar{t})$, $C_{T\,L} = C_T / (1 + \bar{t})$, $\bar{t} \approx 1.23$

Table 3.

Wake Form Approximation at Hover for Basic Test Data (ref. to Table 1)											
Helicopters		Mi - 4				Ka - 32					
Main Rotor		Single Rotor				Coaxial Rotor ($h / R \approx 0.2$, $\bar{t} \approx 1.23$)					
Axial & Radial Location		y / R = f (K1, K2)		r / R = f (λ , A)		y / R = f (K1, K2)		r / R = f (λ , A)		Rotor
Factors		K1	K2	λ	A	K1	K2	λ	A	
Measurements in Flight Tests		0.0192	0.0639	-	~ 0.86	0.0371 0.0431	0.1028 0.0854	-	~ 0.82 ~ 0.91	UR LR
Approximation 1	C. Koning E.D. Safronov	$K = \bar{\nu}_1 = 0.0439$		-	$1/\sqrt{2} =$ $= 0.707$	$K = \bar{\nu}_1 = 0.0628$		-	-	UR LR
2	V.S. Vozhdaev								0.671 0.742	UR LR
3	V.E. Baskin V.S. Kaplan	-	-	-	0.667	-	-	-	-	-
4	A.J. Landgrebe	0.014	0.058	0.249	0.78	0.0156	0.0632	-	-	UR LR
5	J.D. Kocurek J.L. Tangler	0.0173	0.0585	0.248	0.78	0.0381	0.0861	-	-	UR LR
6	V.I. Shaidakov	0.0239	0.0444	0.248	0.751	0.0365 ($k_Л = 3$)	0.0646 ($k_Л = 3$)	-	-	UR LR
7	Proposed method	Kocurek & Tangler Approximation			Flight Test	Kocurek & Tangler Approximation			Momentum Theory	
		0.0173	0.0585	0.248	0.86	0.0381	0.0861	0.356	0.819 0.908	UR LR
Notes		r / R = A for (y / R → ∞) & (ψ → ∞) ; Vozhdaev's approximation for single rotor at (h / R = 0) is coincident with Koning & Safronov one.								

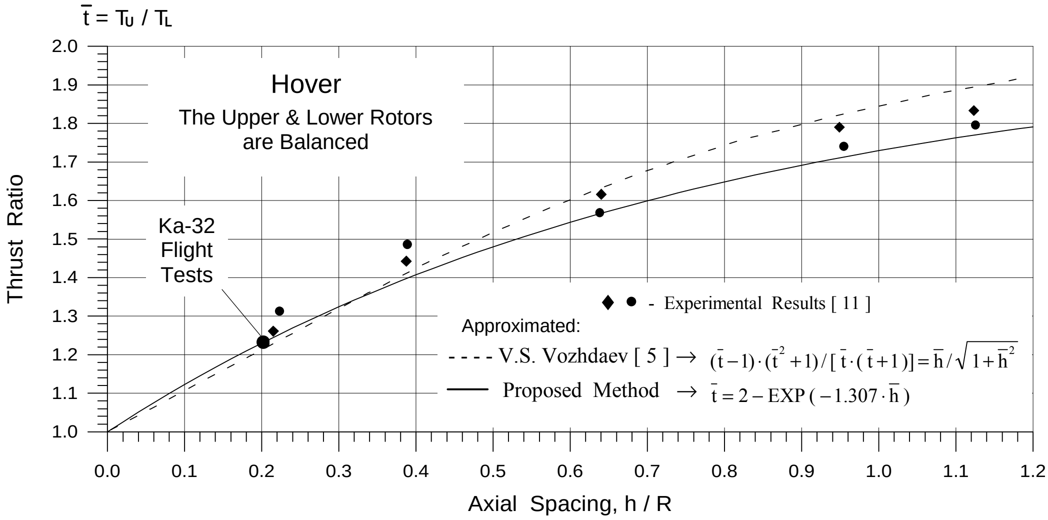


Fig.1. Thrust Ratio Between Upper & Lower Coaxial Rotors

MOMENTUM THEORY

Coaxial rotor ideal power:

$$L_{iC} = L_{iU} + L_{iL} = T_U \cdot V_{1U\,av} + T_L \cdot V_{1L\,av}\,. \quad (1)$$

Shaft torque: $M_K = L/\omega$; $M_{KU} = M_{KL}$ (rotors balanced),

where: $T_U \cdot V_{1U\,av} = T_L \cdot V_{1L\,av}$. (2)

Jet continuity condition: $\begin{cases} A_U^2/1 = V_{1U\,av}/V_{2C\,av}\,, \\ A_L^2/1 = V_{1L\,av}/V_{2C\,av}\,. \end{cases}$ (3), (4)

where: $A_L^2 / A_U^2 = V_{1L\,av} / V_{1U\,av} = T_U / T_L = \bar{t}$. (5)

Ideal power of EQ single rotor: $L_{iS} = T \cdot V_{1S\,av}$. (6)

Ideal power relation: $L_{iC} / L_{iS} = J_C$,

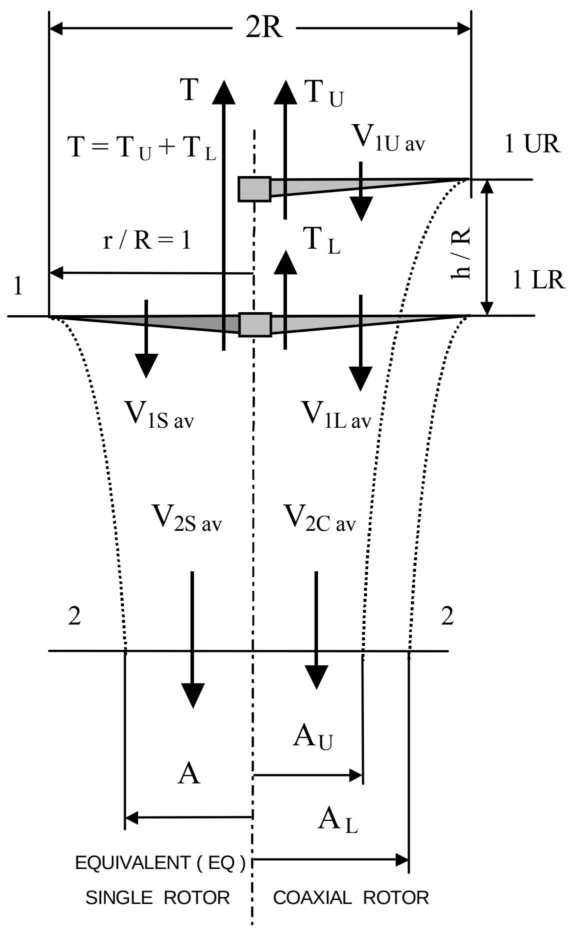
where: J_C – inductions factor of the coaxial rotor.

$$\Rightarrow \underbrace{T\cdot V_{1S\,av}\cdot J_C}_{\text{EQ Single Rotor}} = \underbrace{T_U\cdot V_{1U\,av} + T_L\cdot V_{1L\,av}}_{\text{Coaxial Rotor}} = 2\cdot T_L\cdot V_{1L\,av}\,, \quad (7)$$

where: $T = T_U + T_L$. (8)

Velocities ratio: $V_{1S\,av} = V_{1L\,av}\cdot J_C$; $V_{2S\,av} = V_{2C\,av}/J_C$. (9), (10)

Jet continuity condition: $A^2/1 = V_{1S\,av}/V_{2S\,av}$. (11)



From (1)÷(11):

$$\begin{cases} A_L = A_U\sqrt{\bar{t}}\,; \qquad J_C = \sqrt{2/(1+\bar{t})}\,; \\ A_L = A\sqrt{(\bar{t}+1)/2} \qquad = A/J_C; \\ A_U = A\sqrt{(\bar{t}+1)/(2\cdot\bar{t})} = A/(J_C\sqrt{\bar{t}}). \end{cases} \quad (12)$$

While flight testing of the Mi - 4 → $A \approx 0.86$; Ka - 32 → $\bar{h} \approx 0.2$, $\bar{t} \approx 1.23$		
	Calculation on formula (12) for : $A = 0.86$; $\bar{t} = 1.23$	Flight Tests
A_U	0.819	~ 0.82
A_L	0.908	~ 0.91

Fig.2. Jet Geometry Main Ratios of Coaxial / Equivalent Single Rotors at Hover

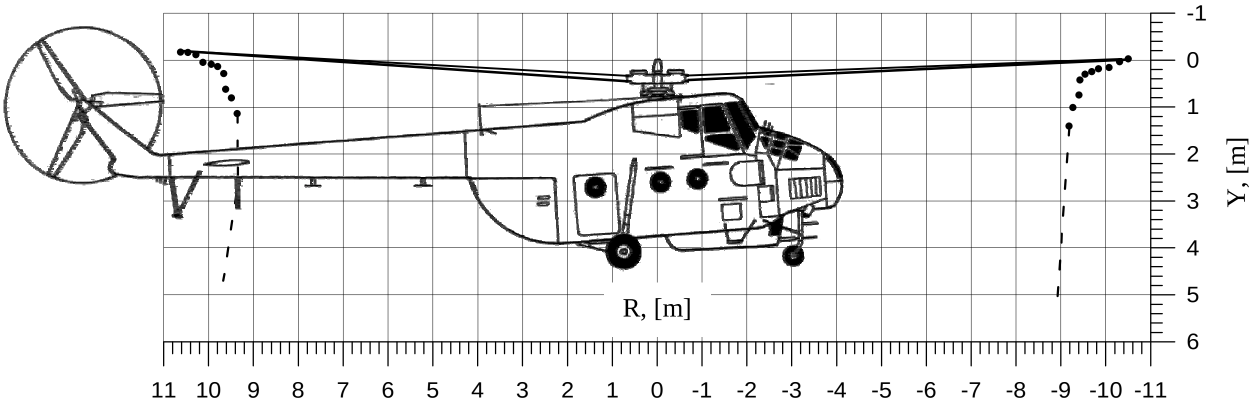


Fig.3. Measured Wake Geometry of Mi-4 Helicopter at Hover
(Shown: tip vortex (•) through Δψ = 10.5° blade turner & flow boundary (- - -)
upon visible smoke fragments. Hover height : H ≈ 500 [m]. Flight Tests 1977)

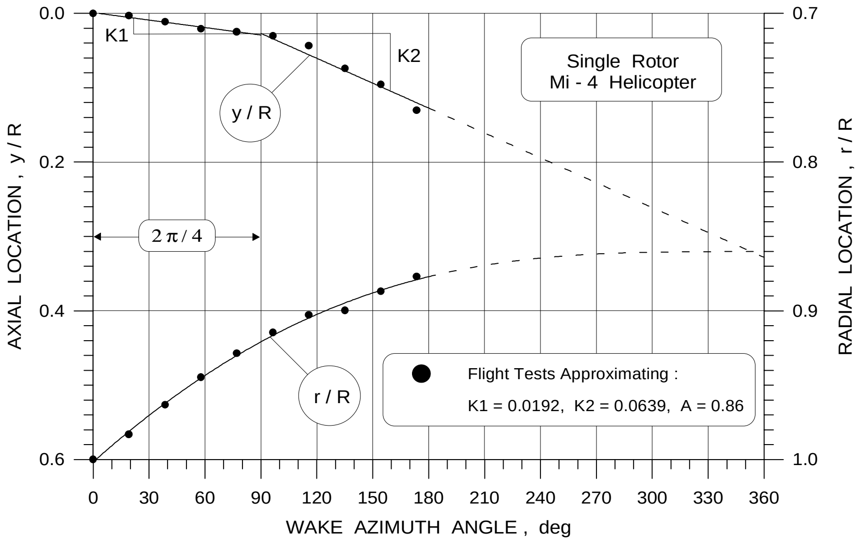


Fig.4. Wake Form & its Approximation at Hover

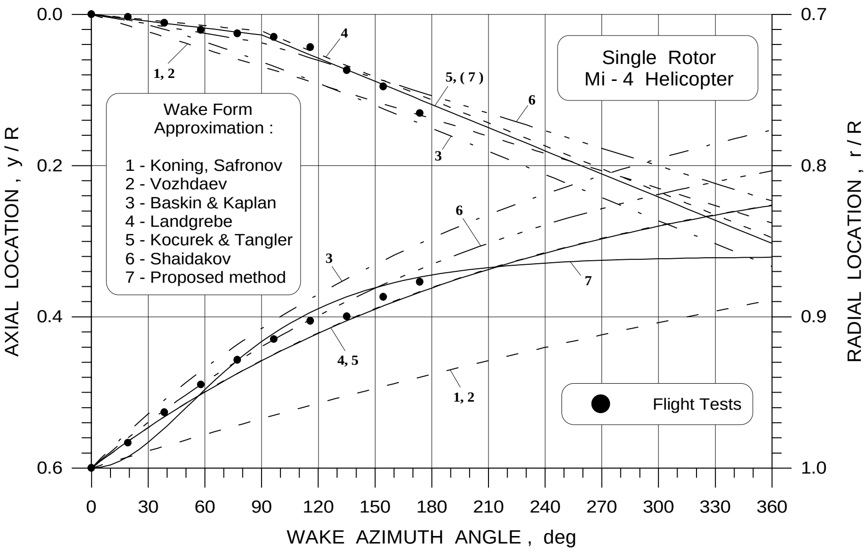


Fig.5. Wake Form & its Approximation at Hover

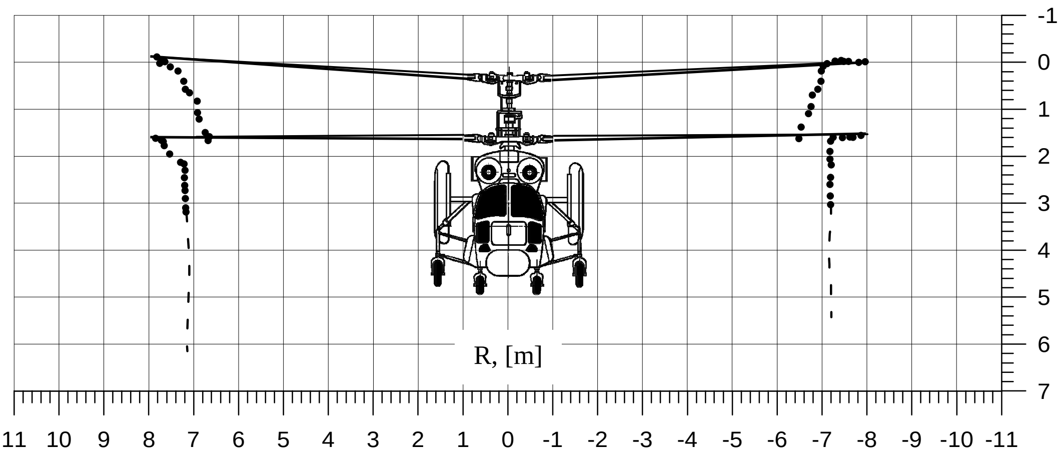


Fig.6. Measured Wake Geometry of Ka-32 Helicopter at Hover
(Shown: tip vortex (•) through Δψ = 15.3° blade turner & flow boundary (- - -) upon visible smoke fragments. Hover height : H ≈ 20 [m]. Flight Tests 1993)

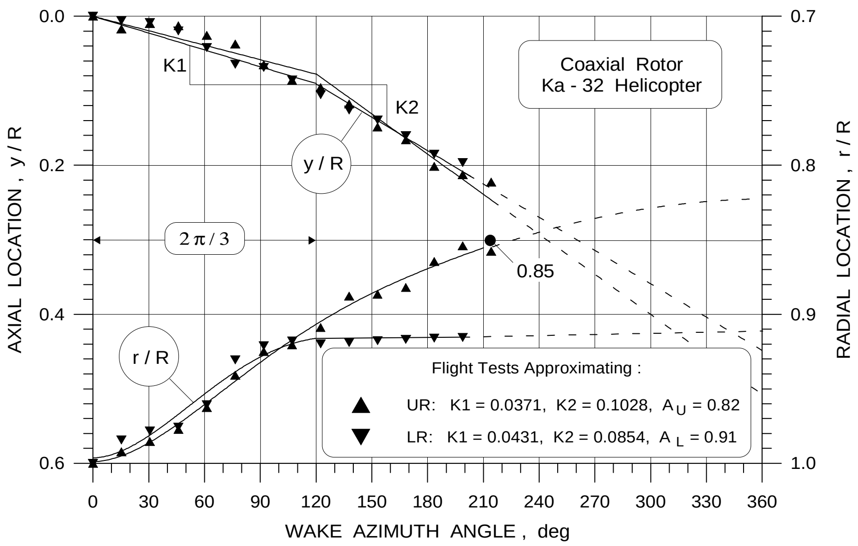


Fig.7. Wake Form & its Approximation at Hover

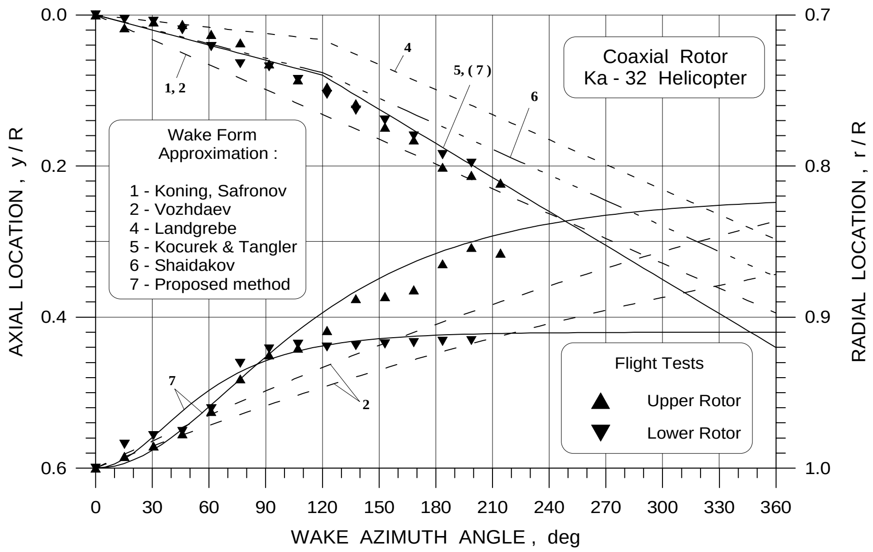


Fig.8. Wake Form & its Approximation at Hover

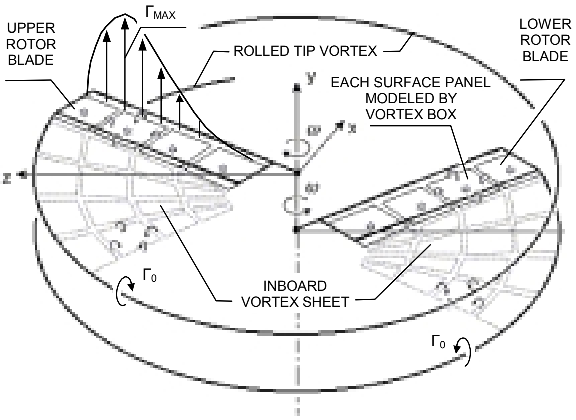


Fig.9. ULISS-6 CODE schematic model of the coaxial rotor wake at hover

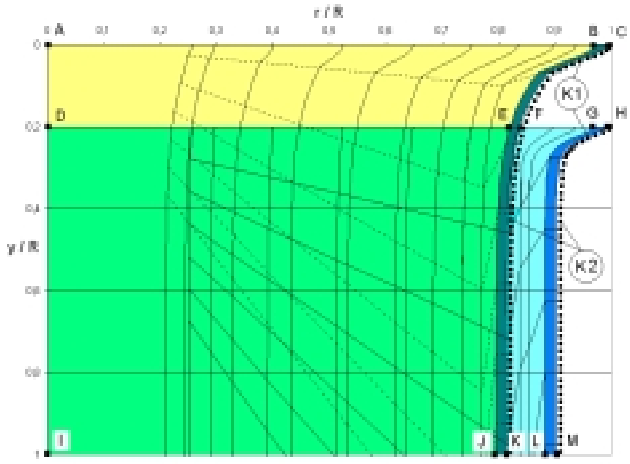


Fig.10. Cross – sectional view (R – Y) of the vortex wake motion geometries of the coaxial rotor at hover as foreseen by using the ULISS-6 CODE

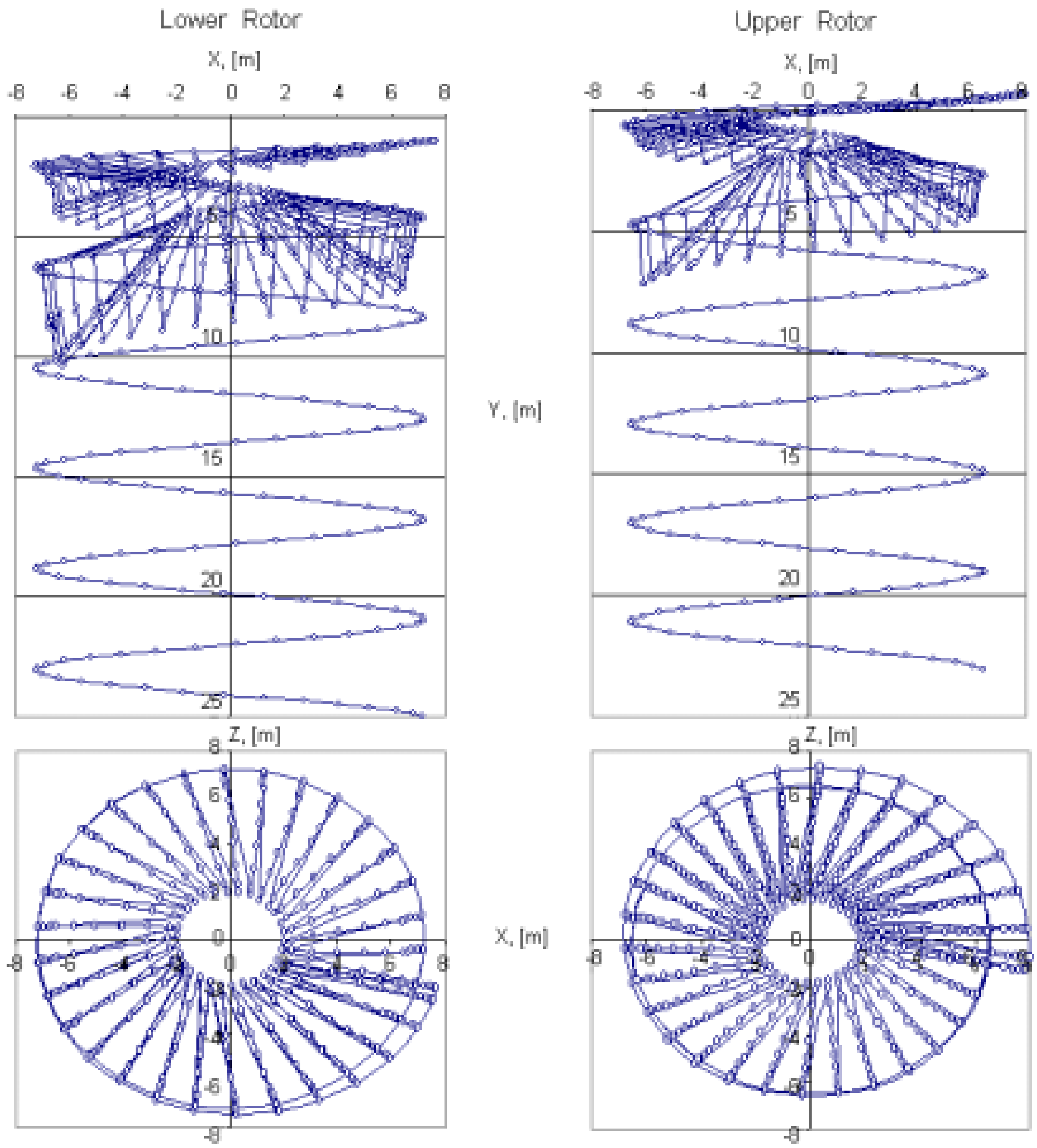


Fig.11. Axial / radial vortex wake motion geometries of the coaxial rotor at hover as foreseen by using the ULISS-6 CODE

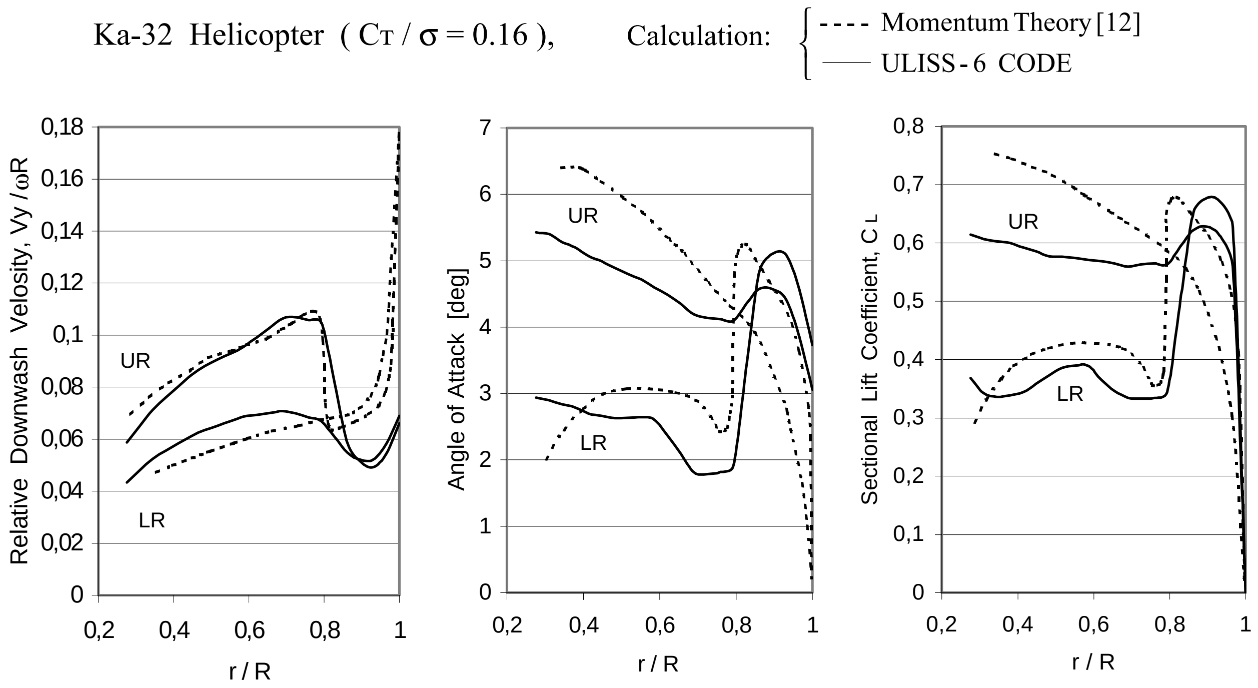


Fig.12. Flow Parameters & Loads Distribution of the Rotor Blade at Hover

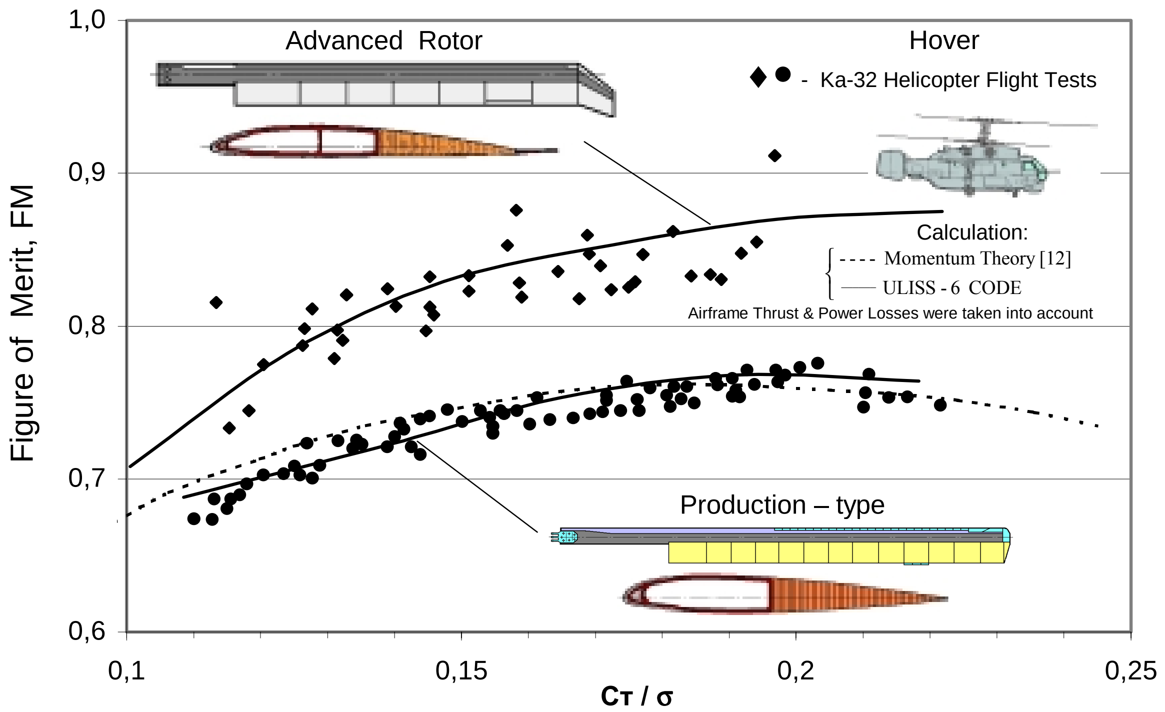


Fig.13. Comparison of Calculated & Flight Tests Figures of Merit