# RADIOMETERS

Black Body Radiation
Infrared Radiometers
Microwave Radiometers

## REVISED EXAM DATE

- SENSORS EXAM
- Monday June 25
- 3pm

# RADIOMETERS

- Instruments for detecting and measuring electromagnetic radiation.
- The term is generally, though not always applied to devices that measure infrared radiation
  - In these lectures we will consider both IR and millimetre wave radiometers

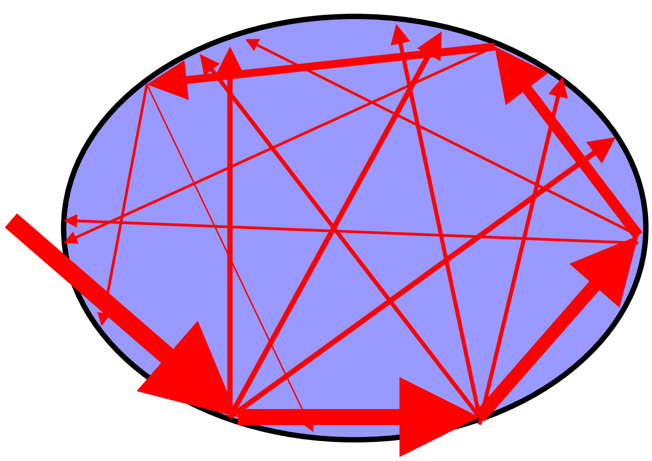
## Thermal Emission

- In an object at a temperature above absolute zero (0 Kelvin or -273.16°C) every atom and every molecule vibrates.
- According to the laws of electrodynamics, a moving electric charge is associated with a variable electric field that in turn produces an alternating magnetic field.
- In essence this vibration produces an electromagnetic wave that radiates from the body at the speed of light.

# Blackbody Radiation

- A blackbody is defined as an object that absorbs all radiation that impinges on it at any wavelength.
- Kirchoff's Law states that any body that is capable of absorbing all radiation is equally capable of the emission of radiation
- An example of a blackbody radiator is a lightproof box pierced by a small hole
  - Any radiation that enters the hole is scattered and absorbed by repeated reflection from the walls so that almost no energy escapes.
  - Such a box when heated becomes a cavity radiator that radiates energy the characteristics of which are determined only by the temperature of the interior walls of the box.

# Blackbody Absorption



## Planck's Function

- Planck proposed that the frequency of the radiation emitted, and hence the energy should be quantised to specific values determined by the frequency

$$E = hf = hc/\lambda$$

where: $E$ – Energy (J)
$h$ – Planck's constant (6.625x10$^{-34}$ Js)
$f$ - Frequency (Hz)
$c$ – Speed of Light (m/s)
$\lambda$ - Wavelength (m)

- He derived (empirically) a formula to relate the radiated power spectral density from a blackbody radiating into cold space at any temperature

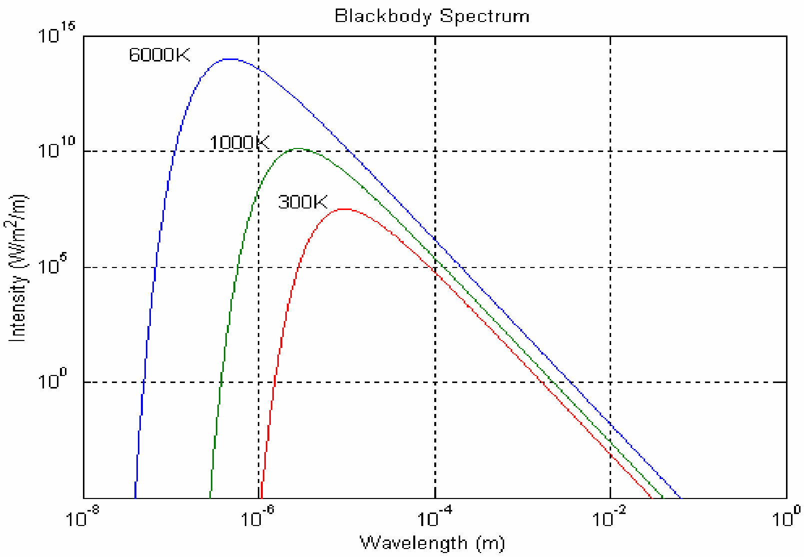
## Planck's Equation

$B_\lambda(T)$ is the energy in Joules emitted per second per unit wavelength from one square meter of a perfect blackbody at a temperature $T$ (Kelvin) into cold space

$$B_\lambda(T) = \frac{2\pi hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

where: $h$ – Planck's Constant (6.625x10$^{-34}$ Js)
$k$ – Boltzmann's Constant (1.3804x10$^{-23}$ J/K)
$\lambda$ – Wavelength (m)
$c$ – Speed of light (3x10$^{8}$ m/s)
$T$ – Temperature (K)

## Blackbody Spectra



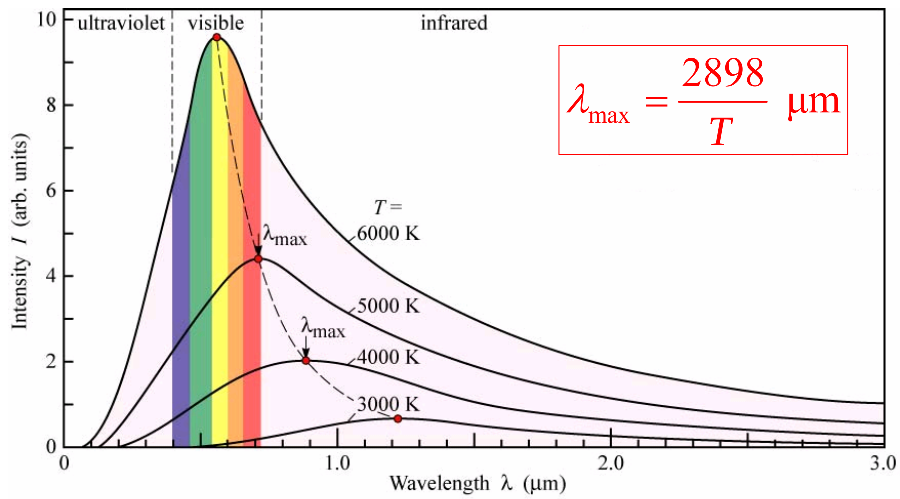

## Properties of the Planck Function

- Even though the radiation energies are quantised, there are so many that they form a continuous spectrum that extends right down into the microwave band
- The most probable frequency is determined by equating to zero the first derivative (with respect to $\lambda$) of Planck's Equation to give.

$$\lambda_m = \frac{2898}{T} \quad \mu\text{m}$$

- This is known as Wein's Law. It states that the higher the temperature, the shorter the peak radiated wavelength

## Wien’s Displacement Law




## Wien’s Displacement Law

## Examples

| Temperature (Kelvin) | Wavelength ($\mu$m) | Frequency Band |
|---|---|---|
| 6000 (the sun) | 0.48 | Visible (blue-green) |
| 310 (human) | 9.35 | Far Infrared Band |
| 4.2 (cosmic background) | 690 | Terahertz Band |

## Rayleigh-Jean Law

- At long wavelengths where the photon energy is small compared to the thermal contribution ($hc/\lambda << kT$), the power density per unit wavelength is proportional to temperature.
- This can be derived by expanding the denominator of Planck's Equation using the Taylor series and setting the higher order terms to zero as they will be very small

$$e^x - 1 = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \approx x$$

$$e^{hc/\lambda kT} - 1 \approx \frac{hc}{\lambda kT}$$

$$B_\lambda(T) = \frac{2\pi hc^2/\lambda^5}{e^{hc/\lambda kT} - 1} \approx \frac{2\pi hc^2}{\lambda^5}\frac{\lambda kT}{hc} = \frac{2\pi kc}{\lambda^4}T$$

## Total Power Density

- The total power density W/m$^2$ within a given bandwidth is determined by integrating $B_\lambda$ over that bandwidth
- As there is no closed form, this is normally solved numerically or by approximation

$$\Phi^o = \int_{\lambda_1}^{\lambda_2} \frac{2\pi hc^2/\lambda^5}{e^{hc/\lambda kT} - 1} d\lambda \quad \mathrm{W/m^2}$$

- A closed form solution does exist for the integral of the Rayleigh-Jean form of the equation if the bandwidth is well away from the peak

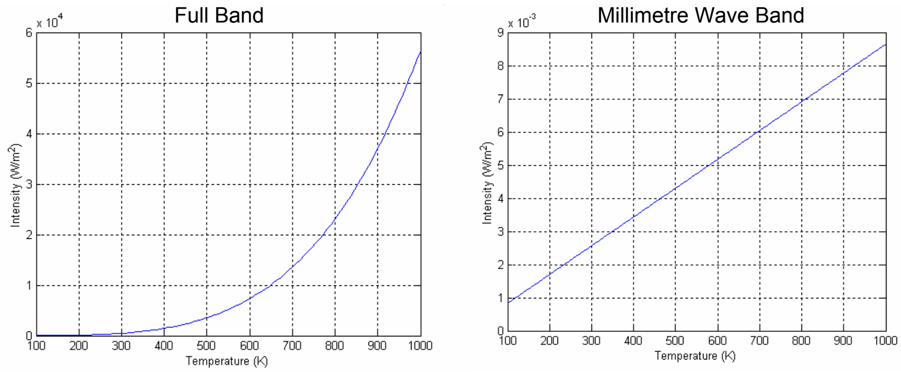
## Stefan-Boltzmann Law

- If the bandwidth includes much more than 50% of the total radiated power, then the Stefan-Boltzmann Law can be used to approximate the radiated power density, $\Phi^o$

$$\Phi^0 \approx \sigma T^4 \quad \mathrm{W/m^2}$$

Where $\sigma$ - Stefan-Boltzmann Const. (5.67x10$^{-8}$ W/m$^2$/K$^4$)

## Examples: Total Power Density




- Integration over the full band shows the classic $T^4$ relationship, while integration over the millimetre wave band shows a linear relationship with temperature.
- The amount of power in the millimetre wave band is smaller by a factor of $10^7$ than the total power over the full band even at low temperatures

## Emissivity and Reflectivity

- Different materials absorb reflect radiation by different amounts
  - Metals reflect most of the incident power
  - Dark, rough materials (like soot) absorb most of the incident energy and get warm
- Metals have free electrons in their conduction band that are free to oscillate in unison in response to the electric field of an incoming EM wave
- By oscillating they radiate (like an antenna), and by doing so they reradiate most of the incoming energy (which we call reflection)
- Soot, also has free electrons, but as they oscillate they bump into the lattice (short mean free path), and in so doing cause it to vibrate more vigorously (heat up), and so less power is radiated as EM radiation

## Definitions

- **Emissivity ($\varepsilon$):** A measure of the ability of a body to radiate energy. It is given by the ratio of the power radiated by the body (per unit area) to the power radiated by a blackbody at the same temperature
- **Reflectivity ($\rho$):** The ratio of the power reflected (per unit area) to the power incident.
- **Kirchoff's Law:** The relationship between emissivity and reflectivity is as follows

$$\rho = (1-\varepsilon)$$

## Emissivity at Millimetre Wave Frequencies

| Material | Emissivity ($\varepsilon$) | Reflectivity ($\rho$) |
|---|---|---|
| Skin | 0.98 | 0.02 |
| Wet soil | 0.95 | 0.05 |
| Paint | 0.94 | 0.06 |
| Heavy Vegetation | 0.93 | 0.07 |
| Dry soil | 0.92 | 0.08 |
| Dry grass | 0.91 | 0.09 |
| Sand | 0.90 | 0.1 |
| Asphalt | 0.83 | 0.17 |
| Cotton cloth | 0.80 | 0.2 |
| Oxidised steel | 0.79 | 0.21 |
| Concrete | 0.76 | 0.24 |
| Polished steel | 0.07 | 0.93 |

Note: Other references have slightly different values for these materials

## Application of the Stefan-Boltzmann Law

- The law is generally written to include the surface area of the object and its emissivity, to produce a value for the total radiated power (or flux) towards an infinitely cold (0K) space

$$\Phi = A\varepsilon\sigma T^4$$

Where $\Phi$ – Total power radiated (W)
$A$ – Surface area ($m^2$)
$\varepsilon$ - Emissivity
$\sigma$ - Stefan-Boltzmann Constant ($5.67 \times 10^{-8}$ $Wm^{-2}K^{-4}$)
$T$ – Temperature (K)

## Effect of Surrounding Objects

- In general, objects don't radiate into space, but are surrounded by other objects with other temperatures and emissivities, so a net flux balance must be calculated.
- For example, in the case of a sensor with a temperature $T_S$ and an emissivity $\varepsilon_S$ looking at a target with a temperature $T_T$ and an emissivity $\varepsilon_T$, the net power that flows into the sensor can be determined as follows

$$\Phi = A\varepsilon_T\varepsilon_S\sigma\left(T_T^4 - T_S^4\right)$$

## Derivation

- Radiated from target toward sensor $\Phi_{TO}$.
- Reflected by sensor $\Phi_{TR} = -\Phi_{TO}(1-\varepsilon_s)$
- Leaving the net flux

$\Phi_T = \Phi_{TO} + \Phi_{TR} = \Phi_{TO} - \Phi_{TO}(1-\varepsilon_s) = \Phi_{TO}\varepsilon_s$

- Equating in terms of target characteristics

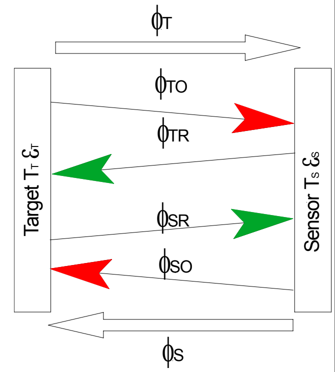
$$\Phi_T = A\sigma\varepsilon_T T_T^4 \varepsilon_S$$

- Similarly the net flux radiating from the sensor toward the target will be

$$\Phi_S = -A\sigma\varepsilon_S T_S^4 \varepsilon_T$$

- Combining the two to obtain the final flux in (W)

$$\Phi = \Phi_T + \Phi_S = A\varepsilon_T\varepsilon_S\sigma\left(T_T^4 - T_S^4\right)$$




## Example (1)

- Calculate the total power that would be radiated by a naked human being in space at absolute zero
  - The surface area ($A$) of a person using Gehan's formula

$$A = 0.0235 \times \text{height}^{0.42246} \times \text{weight}^{0.51456} = 2\text{m}^2$$

for height – 180cm and weight – 80kg

  - The temperature ($T$) of a healthy human being is 37°C (310K)
  - The emissivity ($\varepsilon$) is 0.98 as tabulated

$$\Phi = A\varepsilon\sigma T^4 = 2 \times 0.98 \times 5.67 \times 10^{-8} \times 310^4 = 1026\text{W}$$

  - This power loss would not be sustainable if it were not for the compensating absorption of radiation from surrounding surfaces at room temperature

## What 2m$^2$ of Surface Area can be Used For?

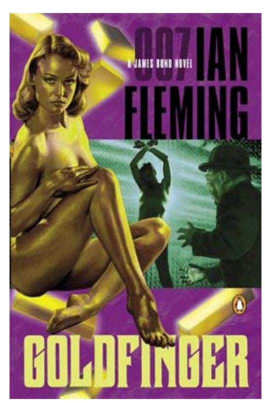

## Example (2)

- In a room with painted walls at a temperature of 20°C the power radiated by the naked human being is as follows

$$\Phi = A\varepsilon_P \varepsilon_W \sigma \left(T_P^4 - T_W^4\right)$$

$\Phi$ = 2x0.98x0.94x5.67x10$^{-8}$(310$^4$ – 293$^4$)
= 195W

- This is a more reasonable amount of power to radiate.
- It is slightly less than the amount of heat generated by a human being



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## Example (3)

- What is the power radiated into cold space over a 2GHz band centred at 100GHz

$$\Phi \approx A\varepsilon_P \int_{\lambda_1}^{\lambda_2} \frac{2\pi kcT}{\lambda^4} d\lambda = \frac{2\pi A\varepsilon_P kcT}{3}\left(\frac{1}{\lambda_1^3} - \frac{1}{\lambda_2^3}\right)$$

- For $\lambda_1$ = 3.03x10$^{-3}$ m and $\lambda_2$ = 2.97x10$^{-3}$ m, the total radiated power equates to 11.7x10$^{-6}$ W
- This is a very small amount of power
- However, because everything else appears cold in this band, it is not so difficult to detect

## DETECTING THERMAL RADIATION

- Three distinct interactions that can be exploited to detect thermal radiation
  - External Photoeffect
  - Internal Photoeffect
  - Heating

## The Atomic Model

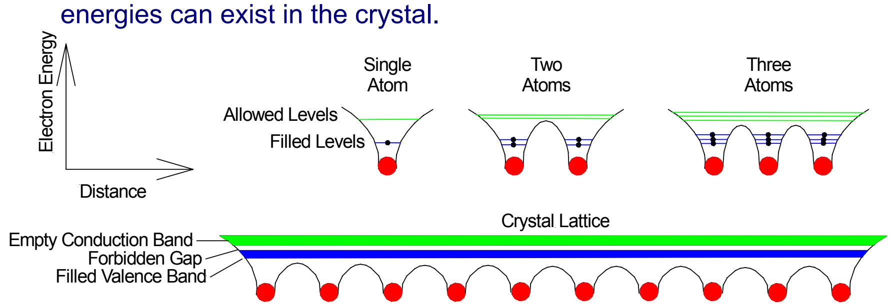
- Positively charged nucleus
- Surrounded by clouds of negative electrons in "orbit"
- The potential energy of the electrons is proportional to their distance from the nucleus
- Not all energy levels are allowed so electrons are found only at discrete stable *energy levels*
- Pauli's exclusion principle restricts the number of electrons at each energy level

## Energy Bands

- The properties of any solid material, including semiconductors, depend upon the nature of the constituent atoms, and upon the way that the atoms are grouped together.
- When atoms come together to form a solid crystal, the outer or *valence* electrons interact to bind the atoms together.
- Because these electrons are shared by more than one atom in the crystal, the energy levels allowed, take on many more closely spaced states.
- This large number of discrete but closely spaced energy levels is called an *energy band*

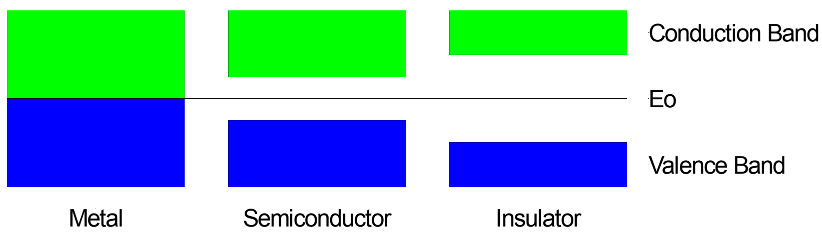
## Energy Bands

- The lowest energy band called the *valence band* is completely filled with electrons for which there is one electron for each of the available energy levels.
- Conversely, the upper energy band is empty of electrons because it corresponds to the unoccupied higher energy levels of the individual atoms. This is called the *conduction band*.
- The energy level between the valence band and the conduction band is called the *forbidden energy gap* since no electrons with such energies can exist in the crystal.



## Energy Bands in Insulators, Semiconductors & Metals

- Metals have no forbidden gap (overlapping valance and conduction bands)
- Semiconductors have a small forbidden gap (Ge = 0.72eV)
- Insulators have a large forbidden gap (diamond = 6eV)




## The Fermi Level *Eo*

- At absolute zero, all of the electrons fill up the lowest energy levels and the highest filled level is called the *Fermi level* ($E_o$)
- At higher temperatures some electrons are excited to levels above the Fermi level, consequently a few levels below the Fermi level are partially empty and those above are partially filled.
- In this situation the Fermi level represents the energy at which the levels are half filled and half empty.
- It is indicated on the diagram as $E_o$.

## The Work Function

- At absolute zero, the free electrons in a metal are distributed amongst a large number of energy states (as shown in an earlier slide) up to the *Fermi level* ($E_o$)
- The *work function* of the metal is the energy which must be supplied to free electrons possessing energy $E_o$ to enable them to escape from the metal

## Insulators

- For materials with a large forbidden gap, the energy that can be supplied to an electron from an applied electric field is too small to carry the electron from the valence band up through the forbidden gap into a vacant band, this makes conduction impossible and the material is an insulator.
- In a diamond crystal the forbidden gap extends for about 6eV.
- Conduction cannot occur in the valence band because, when an electron moves in response to an electric field it must gain kinetic energy, consequently it must find an available and allowed empty level at a higher energy to do so. If none are available, it cannot gain the energy and so it cannot conduct.

## Metals (Conductors)

- A solid which contains a partially filled band structure is called a metal.
- Under the influence of an applied electric field, electrons may acquire additional energy and move into higher states. Since these mobile electrons constitute a current, this substance is a conductor, and the partially filled region, the conduction band.
- From an energy band perspective, a metal has overlapping valence and conduction bands.

## External Photoeffect

- If light with an energy $E=hf$ (Joules) that exceeds the work function ($W$) falls on a metal surface, some of the photons will transfer their energy to electrons which will be ejected from the metal.
- The excess energy in Joules $E = hf-W$ will be transferred to the electrons as kinetic energy
- This relationship is known as the Einstein relation and its experimental verification helped to establish the validity of quantum theory
- The energy of the electrons depends on the frequency of the light, while the intensity of the light determines the rate of emission (or current)

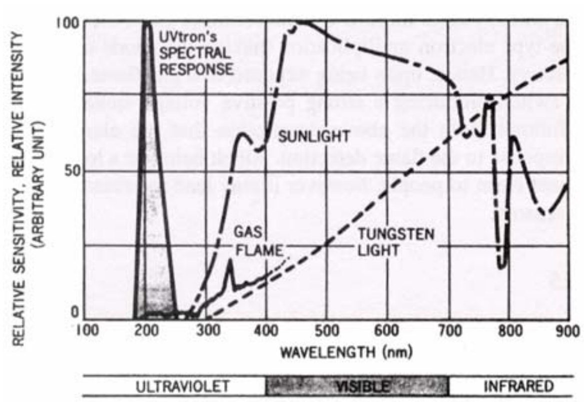
## Work Functions of Some Metals

| Metal | Work Function (eV) |
|---|---|
| Caesium | 1.9 |
| Potassium | 2.2 |
| Sodium | 2.3 |
| Barium | 2.5 |
| Copper | 4.5 |
| Tungsten | 4.5 |
| Silver | 4.6 |

1eV = $1.6 \times 10^{-19}$ J

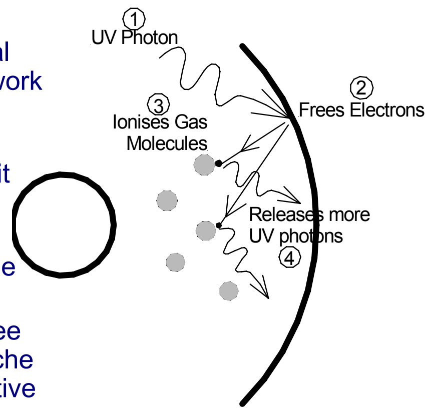
## Example: The UVtron

- Detects missile plumes or gas flames as both produce spectra that extend into the UV with a wavelength of below 200nm
- Sunlight after passing through the atmosphere loses a large portion of its UV spectrum below 250nm

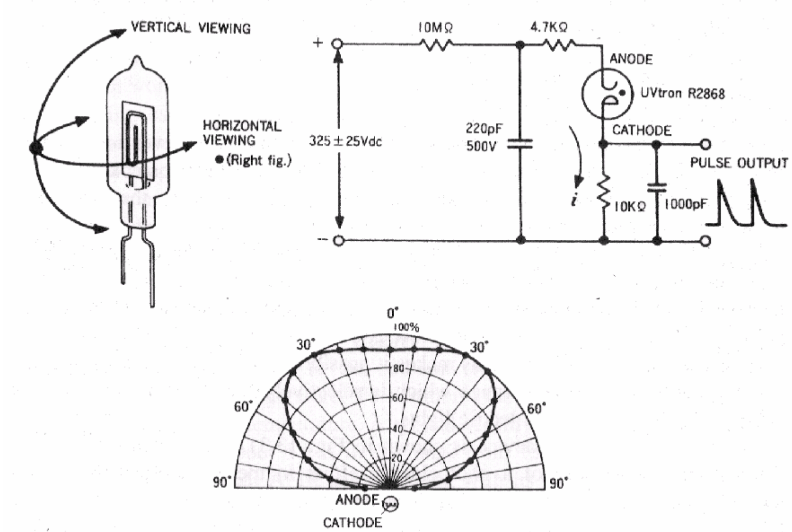


## The UVtron Explained

- The UVtron is a transparent gas filled tube with a high voltage applied across its electrodes
- When a UV photon strikes the special cathode with the appropriately high work function (W), an electron is freed.
- The electron is accelerated by the electric field towards the anode until it strikes one of the gas molecules
- If the electron has gained sufficient energy it ionises the molecule with the release of more UV photons
- These in turn strike the cathode to free more electrons promoting an avalanche effect and the tube becomes conductive
- A relaxation oscillator built around the tube produces a pulse train in the presence of UV




## Relaxation Oscillator for the UVtron

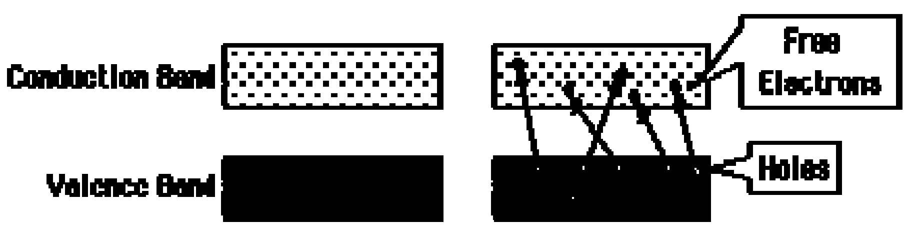


## Internal Photoeffect

- In this effect the photon has sufficient energy to create a free electron, a free hole, or an electron-hole pair in the material
- The material is generally a semiconductor

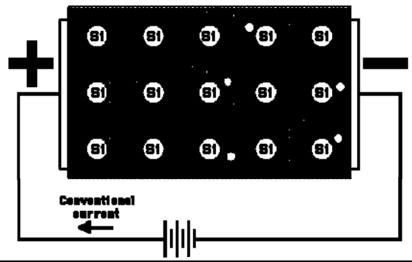
## Semiconductors

- The forbidden gap is comparatively narrow.
- A few electrons can be promoted from the valence band to the conduction band across the forbidden energy gap by virtue of the thermal energy of the crystal at room temperature.
- Semiconductors with a wide forbidden gap are less affected by temperature and hence more desirable



## Photoconductive Detectors

- Electrons promoted into the conduction band can conduct electricity, as can the corresponding vacancies in the valence band (called holes).
- Electrons can be excited into the conduction band by photons with energy larger than the forbidden gap, $E_g$
- Hole current occurs as electrons hop between lattice positions to fill the vacancies left by the freed electrons.
- Since the number of carriers is much fewer than in the case of a metal, semiconductors are poorer conductors than metals




## Band Gaps of Some Semiconductors

| Material | Band Gap Energy (eV) | Long Wave Cutoff $\lambda_c$($\mu$m) | Operating Temp (K) |
|---|---|---|---|
| Si | 1.09 | 1.1 | 300 |
| Ge | 0.81 | 1.4 | 300 |
| PbS | 0.49 | 2.5 | 77 |
| InSb | 0.22 | 5.5 | 77 |
| HgCdTe | 0.025 | 22 | 77 |
| Ge:Hg | 0.087 | 14 | |
| Si:Ge | 0.065 | 17 | |

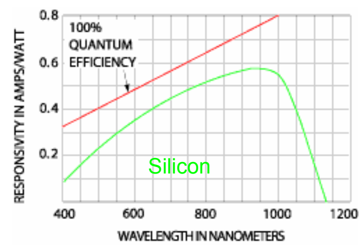
Long Wave cutoff is calculated using the following formula ($E_g$ – Forbidden gap)

$$\lambda_c = \frac{hc}{E_g}$$

## Photovoltaic Detectors

- Photovoltaic effect consists of the generation of a potential difference as a consequence of the absorption of radiation
- The primary effect is photo-ionisation, or the production of hole-electron pairs that can migrate to a region where charge separation can occur.
- This charge separation usually occurs at a potential barrier between two layers of solid material. These can include semiconductor PN junctions and metal-semiconductor interfaces
- For a material with a conversion efficiency $\eta$, the average current (amps) produced by a light beam with optical power, $P$ is as follows

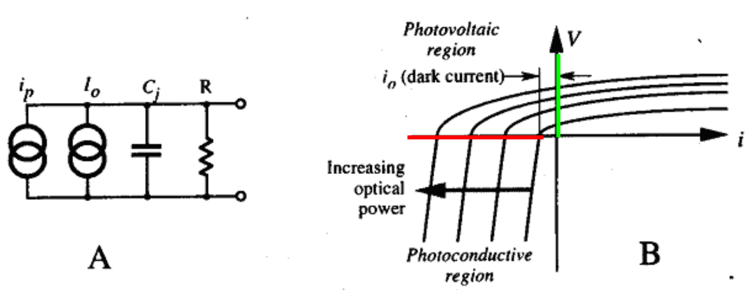
$$i = \frac{\eta e P}{hf} \quad \mathrm{A}$$




- As the output current is proportional to the input power, this is a *square law* detector

## Photodiode Characteristics

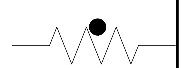
- Can be configured as a *current-to-voltage* converter where the relationship between $P$ and $i_p$ tracks the current axis ($V$=0) (red line)
- Alternatively the diode produces a voltage across its terminals when operated into a high resistance (green line)
- $i_o$ refers to the *dark current* which flows in the absence of any light and is attributed to thermal generation of hole-electron pairs



## Heating Detectors

- Micro Bolometers operate by lattice absorption resulting in increased vibrational energy and hence changes in resistance
  - Metal types have a +ve temperature coefficient of resistance
  - Semiconductor (thermistor) types generally have a –ve temperature coefficient of resistance
- Pyroelectric sensors produce a change in electrical polarisation with changes in temperature
- Golay Cells rely on the expansion of a gas when heated to measure thermal radiation intensity
- Crookes radiometer relies on thermally induced motion of gas molecules to measure radiation intensity
- Thermocouple operation relies on the temperature dependent potential difference that exists between dissimilar metals in contact
- Thermal detectors measure the rate at which energy is absorbed and are therefore insensitive to frequency over a wide range

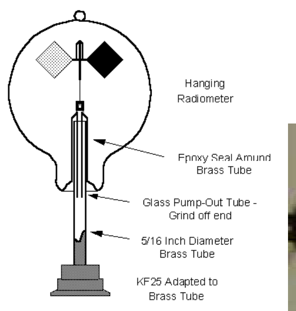
## Thermistors



- Thermistors change their resistance with changes in temperature in a rather exaggerated way.
- Two types: positive temperature coefficient (ptc) and negative temperature coefficient (ntc).
- **ptc thermistors** the resistance increases with increasing temperature (as it does for a pure metal), however, the response is usually extremely nonlinear
- **ntc thermistors**, the resistance decreases with increasing temperature.

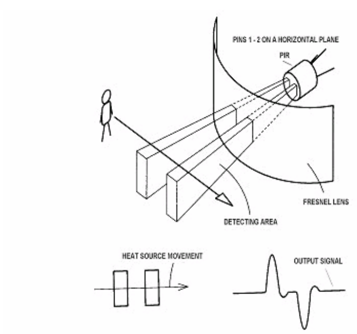
## Crookes Radiometer

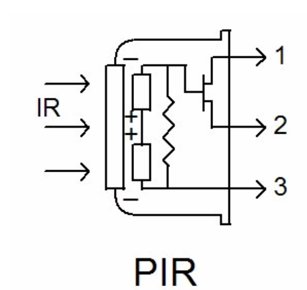
Figure 1 - Radiometer Adaptation




## Passive Infrared (PIR) Sensor

- Best response in the 4-20μm band where human beings radiate
- Often made from a polymer film PVDF that converts a moving hot-spot into an alternating current of the order of 1pA
- A FET follower with an input impedance of 50GΩ converts this to a 50mV signal that can be amplified and detected

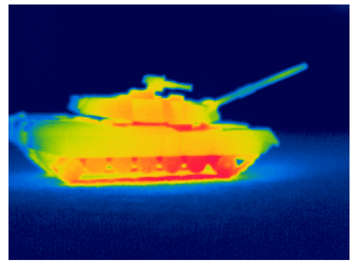


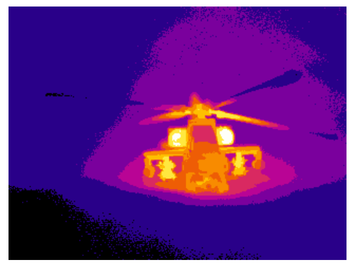



# IMAGING INFRARED

- Electro-optical thermal imagers include the following
  - Forward looking Infrared (FLIR)
  - Thermal imaging systems (TIS)
  - Infrared search and tracking (IRST)
- Generally use the temperature gradient across an object to produce TV like images
- Should not be confused with image intensifiers (night vision), though the boundaries between the two technologies are becoming blurred.

## Example of a Thermal Imager & Some Images made using an Uncooled Sensor



## Thermal Infrared Detectors

- PHOTOCONDUCTIVE DETECTORS
  - Absorb photons to elevate an electron from the valence band to the conduction band of the material, and so change the conductivity of the detector. To detect far IR (8-12$\mu$m) radiation they must be cooled to eliminate the noise generated by thermally generated carriers
- PHOTOVOLTAIC DETECTORS
  - Absorb photons to create an electron-hole pair across a PN junction to produce a small electric current or potential difference
- MICRO BOLOMETERS
  - Absorb thermal energy over all wavelengths, heat up slightly and change their resistance. Do not require cooling.

## Performance Criteria for Detectors

- Responsivity
- Signal to Noise characteristics
- Specific Detectivity (D*)

## Responsivity (R)

- Measurement of how well a detector reacts to incident radiation
- Measured in amps (or volts) per watt of incident radiation

$$R = \frac{I_{sig}}{P(\lambda)A_{\mathrm{det}}} \qquad \text{A/W}$$

where: $R$ – Responsivity (A/W)
$I_{sig}$ – Signal Level (A)
$P(\lambda)$ – Incident power (W)
$A_{det}$ – Detector area ($m^2$)

## Noise Equivalent Power & Detectivity

- Noise equivalent power ($NEP$) is the power incident on the detector that will produce a signal to noise ratio of one

$$NEP = \frac{I_{noise}}{R} \qquad \text{W}$$

- Detectivity ($D$) is the reciprocal of the $NEP$

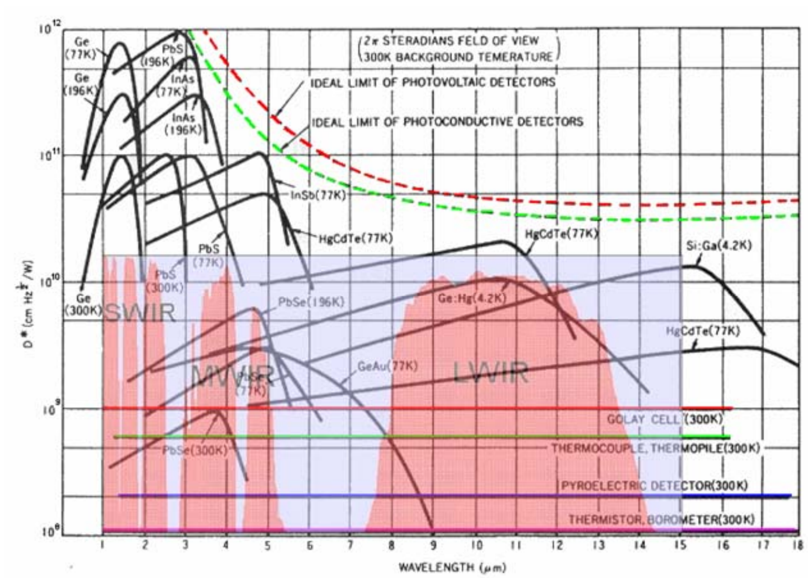
$$D = \frac{1}{NEP} \qquad \text{W}^{-1}$$

## Specific Detectivity ($D^*$)

- Normalised Detectivity because different detectors have different areas and signal bandwidths
- Normalised to a bandwidth of 1Hz and a detector area of 1cm$^2$

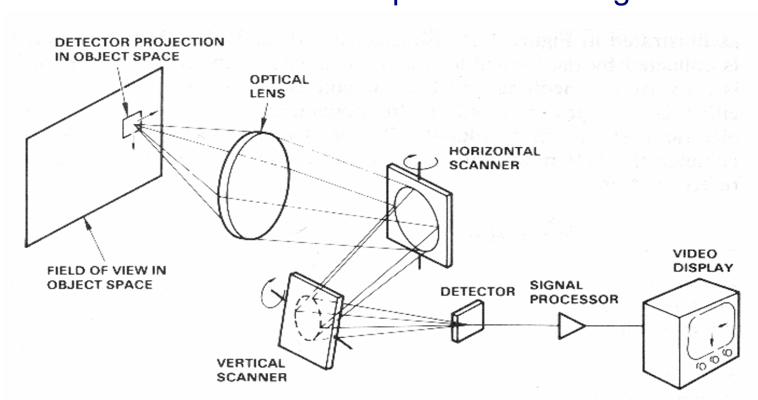
$$D^* = \frac{\sqrt{A_{\det}\Delta f}}{NEP} \qquad \text{cmHz}^{1/2}\text{/W}$$

## Operating Ranges of Some IR Detectors and Transmission Characteristics of the Atmosphere




## Scanning Mechanisms

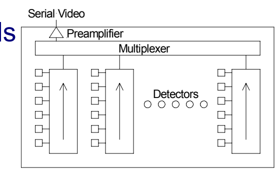
- Early imagers called serial scan sensors relied on a single, or small array of cryogenically cooled elements and a 2D mirror scanner to produce an image

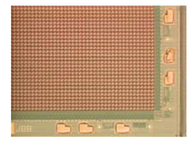


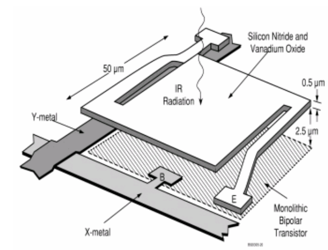

Electro-optical sensors.

## Staring Arrays

- Unscanned 2D staring detector arrays as large as 320x240 pixels
- Constructed on a single sensor chip assembly (SCA) with all supporting electronics
- Can use cryogenically cooled photovoltaic elements made from Mercury Cadmium Telluride (HgCdTe) or Platinum Silicide (PtSi) with a $D^*$ >$10^{10}$
- Can use uncooled micro-bolometer (thermistor) arrays that offer reduced sensitivity but wider bandwidth operation at lower cost with a $D^* = 10^8$





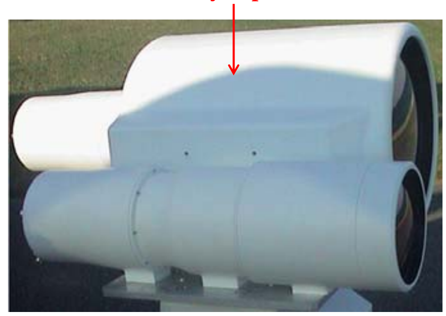

Single Element

## ONR Sponsored MWIR FLIR Sensor

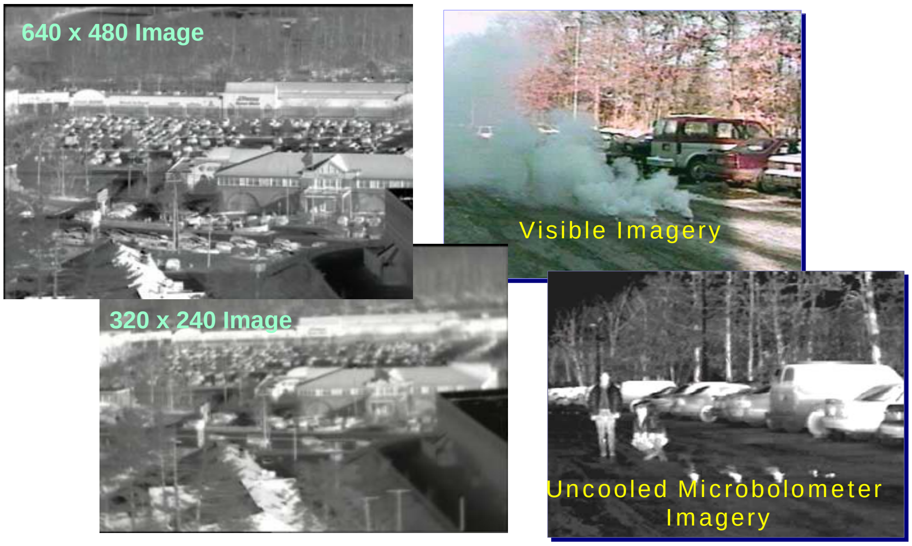
**SENSOR:**

- MWIR InSb 320x240 format sensor
  - high operability
  - high sensitivity
- Dual-FOV 500/1000 mm focal length
  - 1.1° x 0.88° in NFOV
  - 5.5° x 4.4° in WFOV
- f/4.1, 250mm clear aperture
- Sealed housing - sea worthy

Cyclops

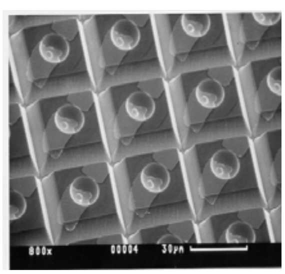


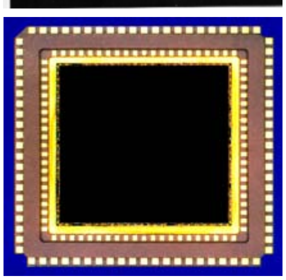
## New Developments: Uncooled LWIR



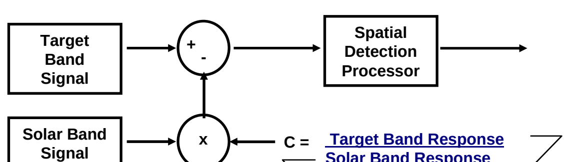

**Long Wavelength Infrared Radiation Penetrates Smoke and Many Obscurants**

## Large Format 2-Color MWIR Focal Plane Array

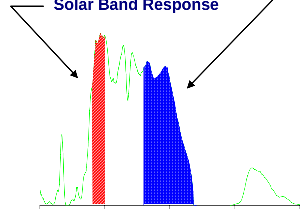


- MWIR/SWIR 2-color 1024x1024 FPA
- Large Dynamic Range Read Out IC (ROIC)
- Gray-Scale Microlenses for High Quantum Efficiency
- MWIR/SWIR for Sun Glint Suppression and Discrimination




- **Solar guard band can be used to remove solar component in target band**
- **Effectiveness depends on correlation between transmission in those two bands**
- **R384 data used to determine bands with best correlation**



## Optical Parameters

- Aperture
- f number
- Focal length
- Detector area
- Transmission effects

## Limiting Aperture Diameter

- The entrance pupil diameter ($D_o$) defines the limiting resolution of the optics as described by the Rayleigh criterion

$$\Phi_{\lim} = \frac{1.22\lambda}{D_o} \quad \text{rad}$$

where: $\Phi_{lim}$ – Smallest angular separation resolvable between two objects (diffraction limited resolution) (rad)

$D_o$ – Aperture diameter (m)

## f Number (f/#)

- Ratio of the focal length and the limiting aperture. It determines the "light" gathering capability and how much energy is focussed on the detector
- Since the light such a lens lets through is proportional to the area of the opening, this would mean if you halve the area, the diameter decreases by $1/\sqrt{2}$.
- f numbers 1, 1.4, 2, 2.8, 4, 5.6, 8, 11, 16 are multiples of $\sqrt{2}$
- To produce an image using the same "light" level, as the f number is increased by 1 stop, the aperture diameter decreases by $\sqrt{2}$ and the exposure time must be doubled
- The scene energy reaching the focal plane is proportional to the reciprocal of the square of the f number for a constant exposure time

## Focal Length (*f*)

- The distance from the optical centre (or pole) to the principal focus of a lens or curved mirror

$$f = (f/\#)D_o$$

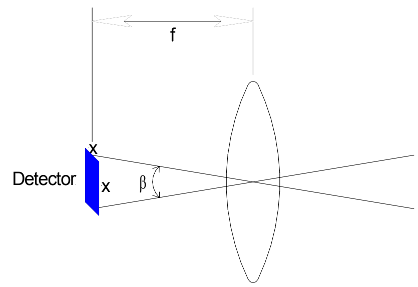
where $f$ – Focal length (m)

$f/\#$ - f number

$D_o$ – Limiting aperture (m)

## Detector Area and Field of View

- The angle of view subtended by the detector of side $x$ is $\beta$
- The relationship between $x$ and $\beta$ is $x = \beta f$ where $f$ is the focal length of the lens
- The field of view in steradians is just $\alpha_d = \beta^2$
- The detector area is $A_{det} = x^2$ which can be written in terms of the field of view and the focal length




$$A_{det} = x^2 = (\beta f)^2 = \beta^2 f^2$$

$$A_{det} = \alpha_d f^2$$

## Transmission Effects ($\tau$)

- Light is lost in an optical system through absorption and reflection
- As the light passes from one refractive medium to another, the portion of the energy transmitted is given by the following

$$\tau = \frac{(1-\rho)^2 e^{-\alpha x}}{1-\rho^2 e^{-\alpha x}}$$

- Where

$$\rho = \left[\frac{(1-n)}{(1+n)}\right]^2$$

- $x$ – Thickness of the lens (m)
- $\alpha$ - Absorption coefficient of the optical material
- $\rho$ - Surface reflectance
- $n$ – Refractive index of the lens material

## Multilayer Coating Lenses

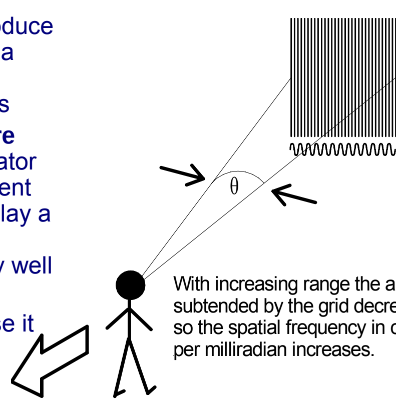
- To increase the transmission of a lens, multilayer coatings are deposited on the lens to grade the refractive index change from air to that of the lens
- For example, uncoated germanium has a transmittance of 47% which can be increased to 97% with careful grading

| Material | Transmission Band (μm) | Refractive Index |
|---|---|---|
| Germanium | 2-20 | 4 |
| Sapphire | 0.4-5 | 1.63 |
| Zinc selenide | 0.5-20 | 2.4 |
| Zinc sulphide | 0.6-15 | 2.2 |

## System Performance Measures

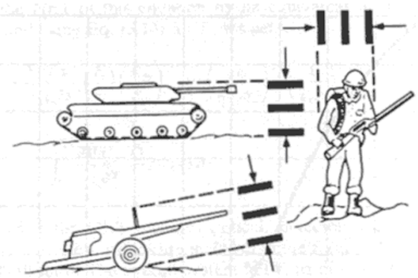
- **Noise Equivalent Temperature Difference (NETD)** Defined as the temperature difference that will produce a signal to noise ratio of unity. It is a measure of the performance of the detector and processing electronics
- **Minimum resolvable Temperature (MRT)** Combines sensor and operator characteristics into one measurement to define the sensors ability to display a structured target with small temperature differences sufficiently well to allow detection or recognition
- MRT can be calculated but because it is statistical in nature it is often determined experimentally

**Spatial Frequency**




With increasing range the angle subtended by the grid decreases, so the spatial frequency in cycles per milliradian increases.

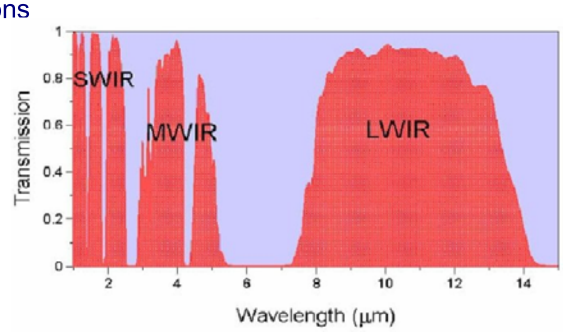
## Target Detection and Recognition



| Target | Resolution per minimum dimension in line pairs | | | |
|---|---|---|---|---|
| Broadside view | Detection | Orientation | Recognition | Identification |
| Truck | 0.90 | 1.25 | 4.5 | 8.0 |
| M-48 tank | 0.75 | 1.20 | 3.5 | 7.0 |
| Stalin tank | 0.75 | 1.20 | 3.3 | 6.0 |
| Centurion tank | 0.75 | 1.20 | 3.5 | 6.0 |
| Half-track | 1.00 | 1.50 | 4.0 | 5.0 |
| Jeep | 1.20 | 1.50 | 4.5 | 5.5 |
| Command car | 1.20 | 1.50 | 4.3 | 5.5 |
| Soldier (standing) | 1.50 | 1.80 | 3.8 | 8.0 |
| 105 Howitzer | 1.00 | 1.50 | 4.3 | 6.0 |
| Average | 1.0 ± 0.25 | 1.4 ± 0.35 | 4.0 ± 0.8 | 6.4 ± 1.5 |

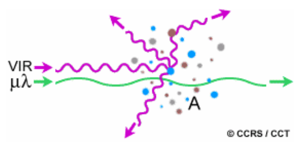
## Clear Air Atmospheric Effects

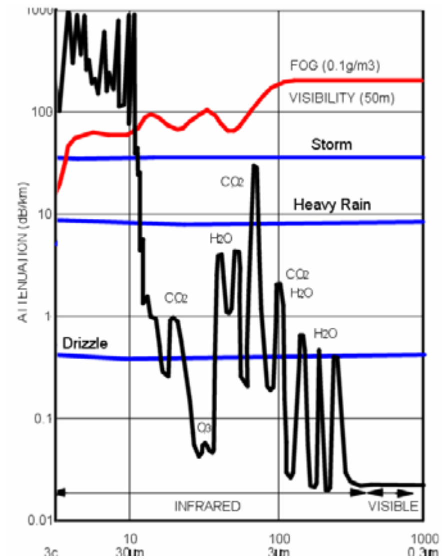
- Radiation is either scattered or absorbed as it propagates through the atmosphere
- A number of “windows” exist in the IR band that are used by systems.
- The most common for missile seekers are MWIR (3-5 $\mu$m) and LWIR (8-12$\mu$m) bands
- For thematic imaging systems, specific bands between 0.45 and 2.35 $\mu$m are selected that offer the best contrast for specific applications



## Bad Weather Effects

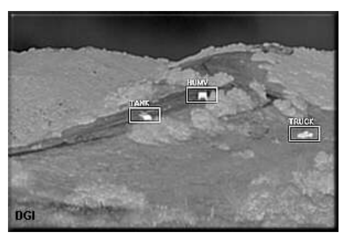
- The effects of heavy fog and rain are shown in the figure
- During clear conditions (visibility > 4km), transmittance losses of up to 1dB/km can occur depending on the humidity
- In rain, transmittance losses are proportional to rain rate
- In mist and fog, transmittance losses are inversely proportional to visibility

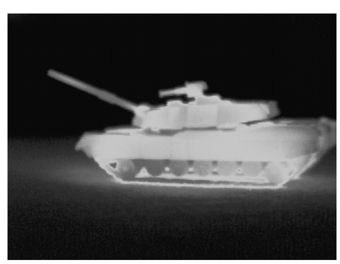



## Example: Tank Detection and Recognition

- Calculate the theoretical detection and recognition ranges for a 3x3m tank with a temperature difference of 2.5°C using the following MRT curve
- Determine the revised ranges taking clear air losses into account
- Determine the revised ranges in mist with a visibility of 1km



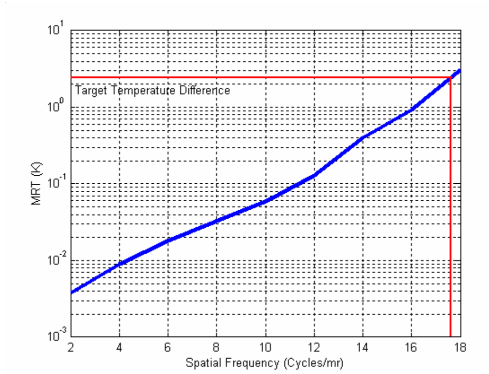



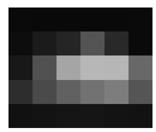
# Theoretical Detection & Recognition Range

- For a temperature difference of 2.5°C, the spatial frequency that can be resolved is 17.6cycles/mr
- From Johnson, for Detection, 1 cycle (2 half cycles) are required
- For Recognition about 4 cycles are required
- For a target with 3m dimensions, each spatial frequency cycle corresponds to a range of 3km
- Therefore

  $R_{det}$ = (3km)x17.6 = 52.8km

  $R_{rec}$ = (3km)x17.6/4 = 13.2km



Detection

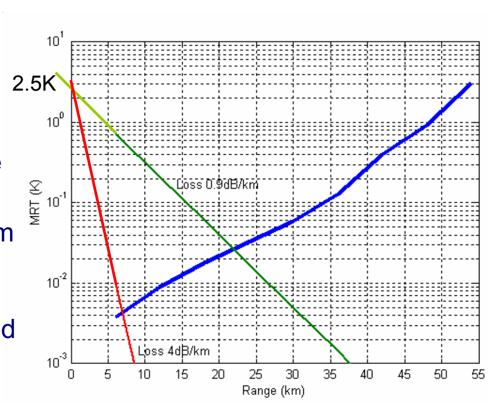
Recognition

# Realistic Detection Range

- Because there is a linear relationship between spatial frequency and range. The graph is scaled so that the detection range scales to the resolvable spatial frequency
- The clear air loss of 0.9dB/km and mist loss of 4dB/km are converted to a transmittance loss $L = 2.5\times10^{\mathrm{dB}/10}$ and plotted on the graph starting at an MRT of 2.5°C until they intersect the MRT curve at the revised detection range

  $R_{det}$ = 22km (clear air)

  $R_{det}$ = 7km (mist)

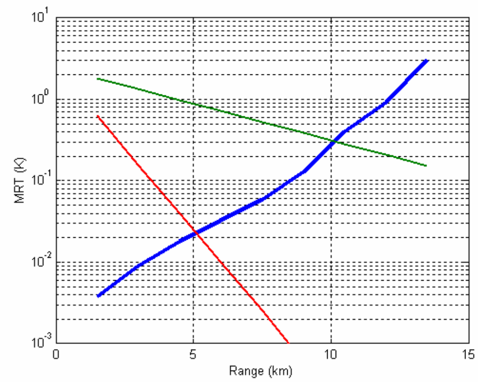


## Realistic Recognition Range

- The MRT graph is now plotted for a recognition range of 13.2km
- The transmittance losses are again plotted

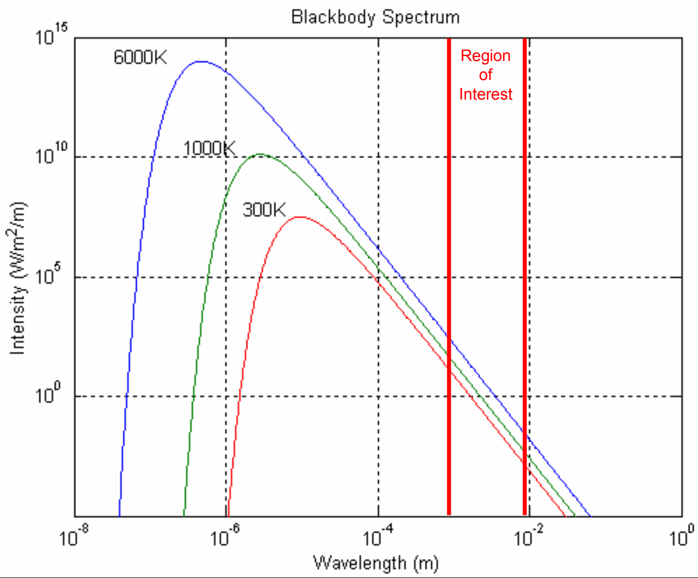
$R_{rec}$ = 10km (clear air)

$R_{rec}$ = 5km (mist)




# Millimetre Wave Receivers and Radiometers

# MILLIMETRE WAVE RADIOMETERS




## Thermal Emission

- The millimetre wave region satisfies the criterion for the Rayleigh-Jean approximation $hc \ll \lambda kT$
- In this band, the power density per unit Hz is generally used

$$B_f = \frac{2kT}{\lambda^2} \quad \text{W/m}^2\text{/Hz/sr}$$

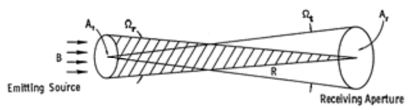
Where $k$ – Boltzmann's Constant ($1.3804\text{x}10^{-23}$ J/K)
$T$ – Temperature (K)
$\lambda$ - Wavelength (m)

## Antenna Power-Temperature Correspondence

- Lossless antenna at a distance $R$ such that the power density $S_t$ is uniform over the solid angle $\Omega_t$
- The power intercepted by the antenna with aperture $A_r$ is just

$$P = S_t A_r = \frac{F_t A_r}{R^2} \quad \text{W}$$

Where $P$ – received power (W)
$S_t$ – target power density (W/m$^2$)
$A_r$ – receiver antenna aperture (m$^2$)
$F_t$ – radiation intensity (W/sr)




## Brightness

- The brightness $B$ is defined as the radiation intensity $F_t$ per unit area

$$B = \frac{F_t}{A_t} \quad \text{W/m}^2\text{/sr}$$

- And the solid angle subtended by the source of the radiation is given by

$$\Omega_t = \frac{A_t}{R^2} \quad \text{sr}$$

- Substituting into the power equation

$$P = BA_r\Omega_t \quad \text{W}$$

## Catering for Antenna Pattern Effects

- For a differential solid angle $\partial\Omega$ we can write

$$\partial P = A_r B(\theta,\phi) F_n(\theta,\phi) \partial\Omega$$

Where $B(\theta,\phi)$ – Source brightness as a function of solid angle (W/m$^2$/sr)

$F_n(\theta,\phi)$ – Normalised radiation pattern of antenna as a function of the solid angle

- Integrating over $4\pi$ steradians and over the frequency band $f_1$ to $f_2$

$$P = \frac{A_r}{2} \int_{f_1}^{f_2} \iint_{4\pi} B(\theta,\phi) F_n(\theta,\phi) \partial\Omega \partial f$$

- This allows us to calculate the power incident on the antenna in terms of the brightness of the source of the radiation and the gain pattern of the antenna

## Antenna Observing a Blackbody

- For the antenna within a blackbody and limiting the bandwidth such that the brightness is constant with frequency, we substitute Rayleigh-Jeans approximation for $B(\theta,\phi)$

$$P_{bb} = \frac{kT(f_2 - f_1)A_r}{\lambda^2} \iint_{4\pi} F_n(\theta,\phi) \partial\Omega$$

- From antenna theory we know that the integral above equates to the pattern solid angle $\Omega_p$ given by

$$\Omega_p = \frac{\lambda^2}{A_r}$$

- Which is substituted back into the power equation to give the fundamental equation of radiometry

$$P_{bb} = kT(f_2 - f_1)$$

## Points to Note

- The detected power is independent of the antenna gain because the source of radiation is extended and uniform
- The equation is independent of the distance from the radiating target
- The temperature of the antenna structure has no effect on the output power (if the antenna is loss free)
- Temperature and power are interchangeable so we can apply all of the gain calculations directly to the measured temperature
- The power detected is directly proportional to the bandwidth

## Brightness Temperature ($T_b$)

- All bodies are to some extent "grey" as they radiate less than a blackbody at the same temperature
- The brightness temperature is defined such that the brightness of the grey body is the same as a blackbody at that temperature

$$T_b(\theta,\phi) = \varepsilon(\theta,\phi)\, T$$

Where $\varepsilon(\theta,\phi)$ – Emissivity

$T$ – Physical temperature of radiating element (K)

## Example

- Determine the brightness temperature of an element at 310K with an emissivity of 0.8
- Determine the power received by an antenna and receiver looking at that element if the receiver bandwidth is 2GHz

$$T_b(\theta,\phi) = \varepsilon(\theta,\phi)T = 0.8\times 310 = 248K$$

$$P = kT_b\Delta f = 1.38\times 10^{-23}\times 248\times 2\times 10^{9} = 6.84\times 10^{-12}W$$

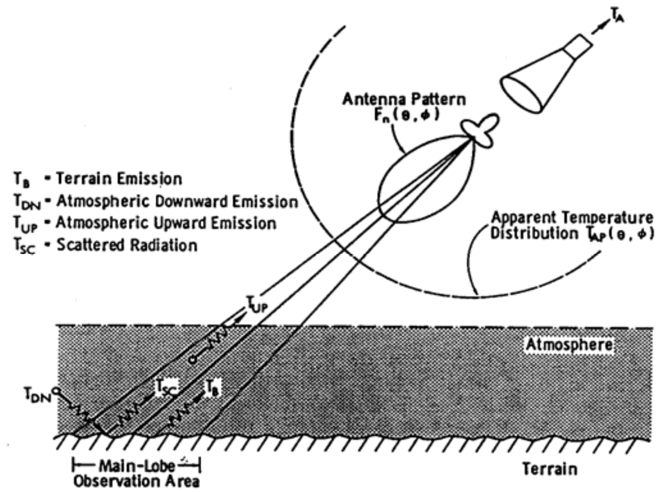
- In general powers are converted to dB relative to 1mW

$$P = 10\log_{10}(6.84\times 10^{-12})+30 = -81.6dBm$$

## Apparent Antenna Temperature

- In radiometry, the apparent temperature measured by the antenna $T_{AP}$ replaces the received power as the measure of signal strength
- $T_{AP}$ is defined as the temperature of a matched resistor with noise power output of $W = kT_{AP}$ at the antenna port (as shown in the following diagram)
- The apparent brightness temperature includes atmospheric losses and the antenna efficiency

## Radiometer Scene Temperatures




## Temperature Components of $T_{AP}$

- $T_B$ Brightness temperature from terrain emissions
- $T_{SC}$ The scatter temperature which is radiation reflected from the terrain into the main lobe of the antenna. This might include components of the atmospheric downwelling temperature $T_{DN}$ and galactic radiation
- These contributors to the total radiation are attenuated by the atmosphere ($L_A$) before they reach the antenna
- $T_{DN}$ Atmospheric downwelling temperature. Depends on the weather
- $T_{UP}$ Atmospheric upwelling temperature. Depends on the weather and how much of the atmosphere lies between the ground and the antenna

$$T_{AP}(\theta,\phi) = T_{UP}(\theta,\phi) + \frac{1}{L_A}\left[T_B(\theta,\phi) + T_{SC}(\theta,\phi)\right]$$

## Atmospheric Attenuation

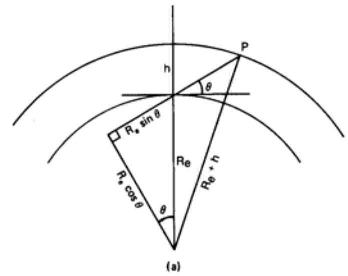
- Attenuation through the atmosphere is a function of air density and must be determined by integration over the path.
- At 94GHz the attenuation through the whole atmosphere at the nadir angle

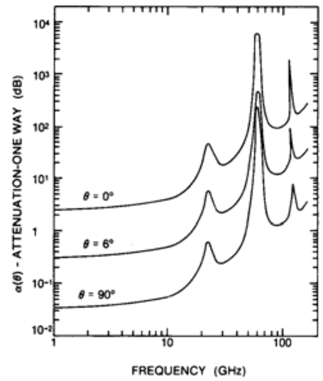
$$L_{AdB} = 0.17 + 0.06\rho_o$$

Where $L_{AdB}$ – Loss (dB)

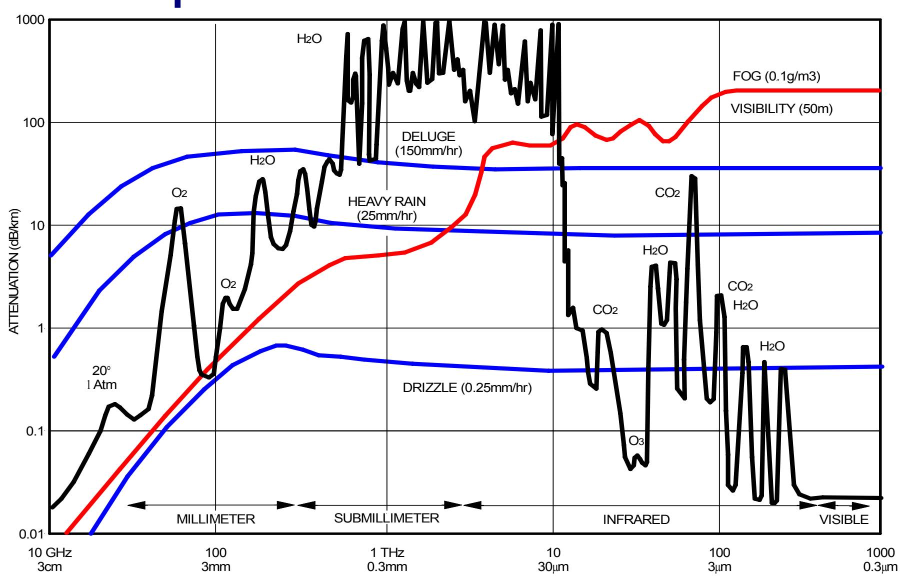
$\rho_o$ – Water vapour conc. (g/m$^3$)

- For aircraft based radiometers the loss in clear air at 94GHz is about 0.2dB/km but it is a function of the local weather conditions

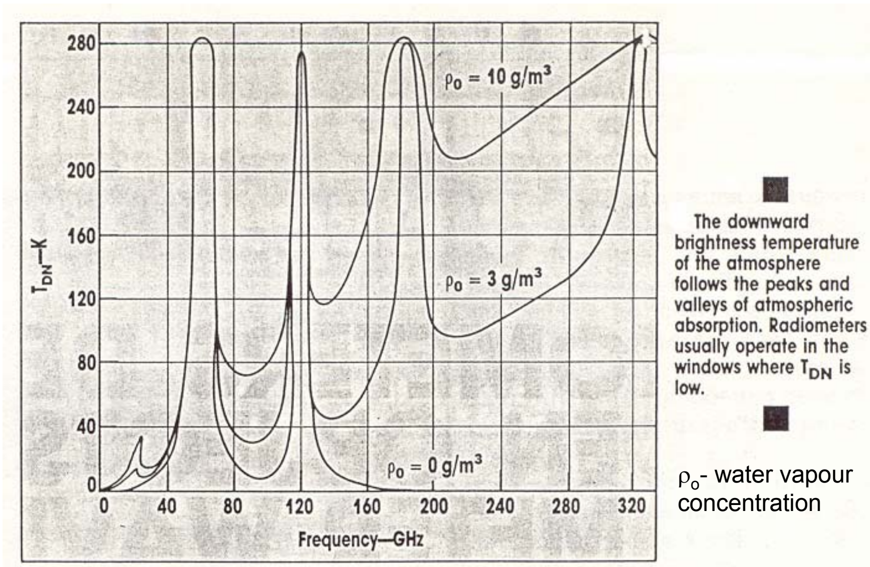



## Atmospheric Loss



## Downwelling Radiation Temperature




$\rho_o$- water vapour concentration

## Upwelling Radiation Temperature

- For space borne radiometers the upwelling radiation is that of the entire atmosphere and is equal to the downwelling radiation
- For aircraft, part of the atmosphere contributes to the upwelling radiation. Under these conditions the atmosphere is modelled as an attenuator, and the upwelling temperature is equal to the effective temperature of the attenuator

$$T_e = (1 - \frac{1}{L_A})T$$

Where $T_e$ – Effective temperature of attenuator (atmosphere)
$L_A$ – Attenuator loss factor $10^{\alpha/10}$
$T$ – Physical temperature of the atmosphere (K)

## Typical Terrain Brightness

- Terrain brightness measurements overlap as they are taken over a variety of weather conditions
  - Metallic Objects: Lossless and opaque with high reflectivities. As a result their brightness will equal the downwelling temperature modified by their reflectivity
  - Water: Depends on polarisation, angle of view and to a lesser extent, temperature, purity and surface conditions. At 94GHz the reported brightness varies between 150 and 300K
  - Soil: Depends on polarisation, angle of view, moisture content and surface roughness (160-280K at 94GHz)
  - Vegetation: Depends on type and moisture content (230-300K at 94GHz)
  - Built-up Areas: Complex, depends on structure etc. Asphalt is given to be 260-300K at 94GHz

## Contrast Temperature

- The table lists typical contrasts that exist between metal objects and terrain types under different weather conditions

| Material | Atmospheric Conditions | | |
|---|---|---|---|
| | Clear | Fog | Rain |
| Vegetation | 200K | 200K | 40K |
| Water | 120K | 100K | 30K |
| Concrete | 190K | 170K | 40K |

## Example

- A space based radiometer operating at 94GHz with a bandwidth of 2GHz looks directly down at the ground at a temperature of 27°C and an average emissivity $\varepsilon$ = 0.9. What is the received power?
- The total loss through the atmosphere (Figure 4.2 Notes) is 1dB. The loss $L_A$ = $10^{dB/10}$ = 1.26
- Assuming the air has a water content of 3g/m$^3$ the downwelling brightness temperature (Figure 4.3 Notes) is 30K. We assume that upwelling and downwelling temperatures are the same
- The reflectivity $\rho$ = (1-$\varepsilon$) = 0.1

$$T_{AP} = T_{UP}(\theta,\phi) + \frac{1}{L_A}\left[\varepsilon T_B + \rho T_{SC}\right] = 30 + \frac{1}{1.26}\left[0.9\times 300 + 0.1\times 30\right]$$

$$T_{AP} = 30 + 216.7 = 246.7K$$

$$P = 30 + 10\log_{10} kT\beta = 30 + 10\log_{10}\left[1.38\times 10^{-23}\times 246.7\times 2\times 10^{9}\right]$$

$$P = 30 - 111.67 = -81.67dBm$$

## Antenna Efficiency

- Up until now it has been assumed that the antenna is lossless, in reality it absorbs some of the power incident on it, and hence it also radiates.
- The apparent temperature measured at the antenna output port $T_{AO}$ is

$$T_{AO} = \eta_1 T_A + (1-\eta_1)T_p$$

Where: $\eta_1$ – Radiation Efficiency of the Antenna (Typ 0.6)
$T_A$ – Scene Temperature measured by the antenna (K)
$T_P$ – Physical Temperature of the antenna (K)

- In this case the efficiency $\eta_1$ is equivalent to the reflectivity

## Antenna Beamwidth

- A typical cassegrain antenna used for a radiometer will have a half-power (3dB) beamwidth given by the following formula.

$$\theta_{3dB} \approx \frac{70\lambda}{D} \quad \text{deg}$$

Where $D$ – Diameter of the antenna (m)

$\lambda$ - Wavelength (m)

- The sidelobe levels of such an antenna will at 10GHz be about -35dB and at 94GHz about -20dB wrt peak

## Beam Fill Ratio (F)

- The size of the antenna footprint does not affect the terrain brightness unless it includes objects of different emissivity (or temperature)
- If the target is totally enclosed within the beam footprint, then the observed brightness temperature $T_B$ can be calculated as follows

$$T_B = T_{BG}(1-F) + T_{BT}F$$

$$F = \frac{A_T}{A}$$

Where $T_{BG}$ – Ground Brightness Temperature (K)

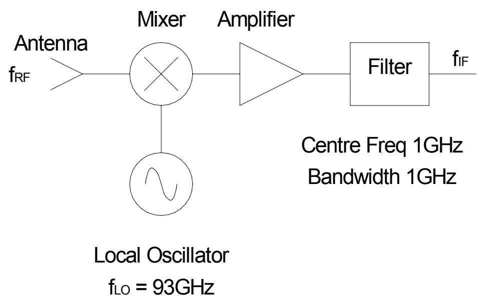
$T_{BT}$ – Target Brightness Temperature (K)

$A_T$ – Target Area ($m^2$)

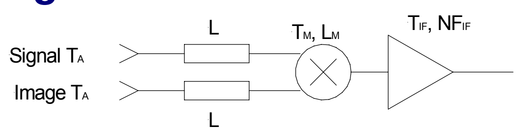
$A$ – Antenna Footprint ($m^2$)

## Radiometer Receivers

- At millimetre wave bandwidths amplifiers are expensive so most radiometers down-convert directly from the antenna port
- To achieve an IF frequency $f_{IF}$=1GHz, the RF frequency $f_{RF}$ could be 92 or 94GHz
- The 92GHz signal is known as the *image* of the 94GHz signal
- As the bandwidth is 1GHz, the receiver measures signals in the band from 91.5 to 92.5GHz and from 93.5 to 94.5GHz




## Noise Figure of a Receiver




- To accommodate both the signal and the image inputs in the analysis, the receiver is drawn with two inputs
- The total noise figure of a cascaded receiver chain is as follows

$$NF = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2}$$

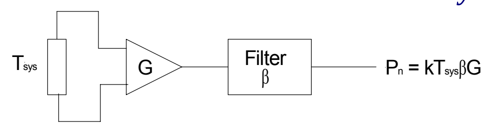
- For this case $L=NF_1$ $L_M/2=NF_2$ $1/L=G_1$ $2/L_m=G_2$ and $NF_{IF}=NF_3$

$$NF_{DSB} = L + L\left(\frac{L_M}{2} - 1\right) + \left(NF_{IF} - 1\right)\frac{L.L_M}{2} = \frac{L.L_M\, NF_{IF}}{2}$$

- For a single sided implementation where the image is filtered out

$$NF_{SSB} = L.L_M\, NF_{IF}$$

## System Noise Temperature $T_{sys}$




- Even in the absence of an external signal, the radiometer will produce an output because the receiver is not at 0K
- This output power $P_N$ is defined in terms of the temperature $T_{sys}$ of a matched resistor at the antenna port and is calculated as follows

$$P_N = kT_{sys}\beta G$$

where: $k$ – Boltzmann's Constant 1.38x10$^{-23}$ J/K
$\beta$ – System Bandwidth (Hz)
$G$ – System Power Gain

## Noise Figure Contribution to Temperature

- The receiver introduces additional noise (over and above the thermal noise) into the system which is incorporated into the equation for $T_{sys}$

$$T_{sys} = T_o(NF - 1)$$

Where $NF$ – Noise figure for the receiver defined as the ratio of the input to output SNR. With $T_o$ = 290K

- Substitute the appropriate noise figure for single or double sided operation into the equation to determine the system temperature

## Example

- Calculate the system noise temperature for the receiver shown in the figure, redo if it is cooled to 77K with liquid nitrogen.
- Convert all the losses etc from dB
  - $L_{WG}$ = 10$^{dB/10}$ = 1.12
  - $L_M$ = 6.31
  - $NF_{IF}$ = 1.78
- Substitute into the DSB formula

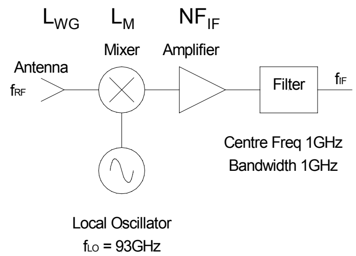
$$NF_{DSB} = \frac{L_{WG}.L_M\,NF_{IF}}{2} = \frac{1.12 \times 6.31 \times 1.78}{2} = 6.29$$

- Calculate $T_{sys}$ for $T_o$ = 290K

$$T_{sys} = T_o(NF_{DSB} - 1) = 290(6.29 - 1) = 1534K$$

- Calculate $T_{sys}$ for $T_o$ = 77K

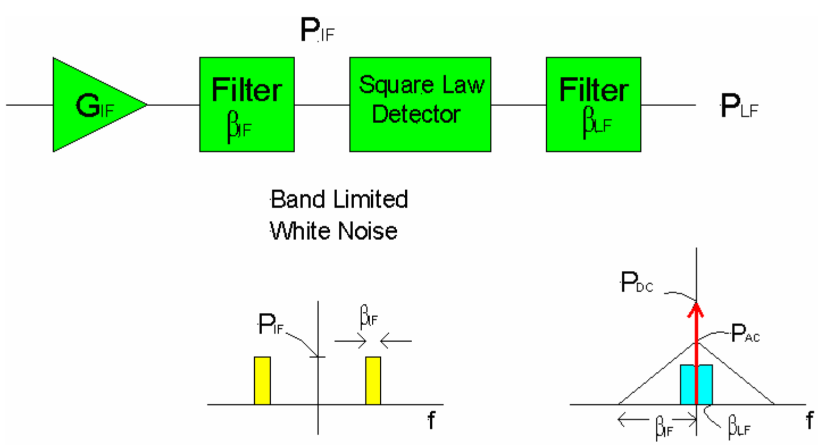
$$T_{sys} = T_o(NF_{DSB} - 1) = 77(6.29 - 1) = 407K$$




$L_{WG}$ – 0.5dB
$L_M$ – 8dB
$NF_{IF}$ – 2.5dB

## Temperature Contrast Sensitivity

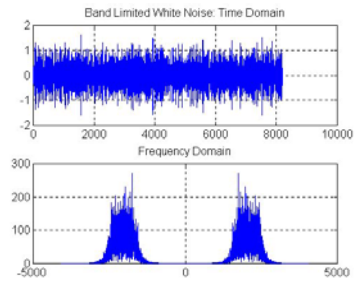
- The ability of a radiometer to detect changes in the input temperature $\Delta T$ is determined from the analysis of the detector output when the input is band limited white noise

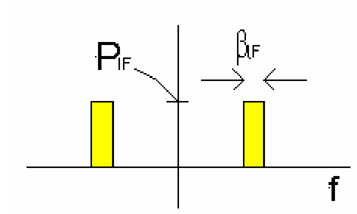


## IF Filter Output

- Assuming a rectangular filter, the double sided spectrum has a bandwidth $\beta_{IF}$ and magnitude

$$P_{IF} = \frac{kTG_{IF}}{2}$$



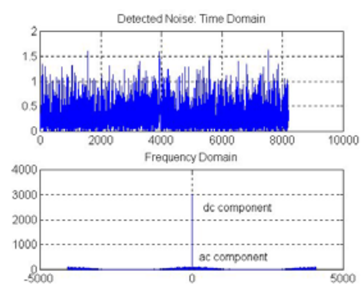

## Square Law Detector Output

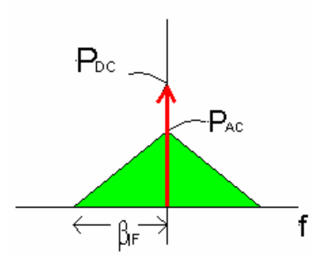
- A square law detector produces an output signal proportional to the square of the input envelope.
- The post detection probability density includes a DC component $P_{DC}$

$$P_{DC} = \left(\frac{kTG}{2}\right)^2 \beta_{IF}^{\ 2}$$

- And a double sided triangular noise component with a magnitude $P_{AC}$ and width $\beta_{IF}$

$$P_{AC} = \left(\frac{kTG}{2}\right)^2 \beta_{IF}$$



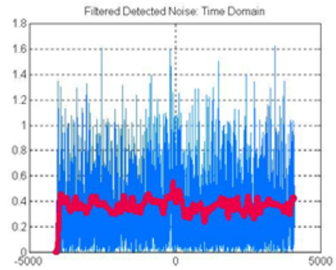
## Final Filtered Output

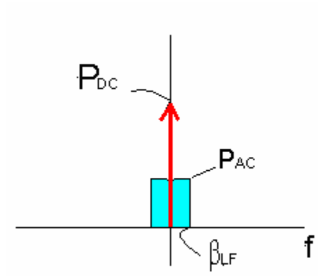
- A lowpass filter with a bandwidth $\beta_{LF}$ reduces the AC component to an almost rectangular density function (because $\beta_{LF}$ << $\beta_{IF}$) with the same height as the unfiltered AC power density function
- The total AC power is equal to the area of the double sided spectrum

$$P_{AC} = 2\left(\frac{kTG}{2}\right)^2 \beta_{IF}\beta_{LF}$$

- The DC component remains unchanged

$$P_{DC} = \left(\frac{kTG}{2}\right)^2 \beta_{IF}^{\ 2}$$




## Detection(1)

- The ratio of the AC power component to the DC power component

$$\frac{P_{AC}}{P_{DC}} = \frac{2\left(\frac{kTG}{2}\right)^2 \beta_{IF}\beta_{LF}}{\left(\frac{kTG}{2}\right)^2 \beta_{IF}^{\ 2}} = \frac{2\beta_{LF}}{\beta_{IF}}$$

- Which in terms of voltages is

$$\frac{V_{AC}}{V_{DC}} = \sqrt{\frac{2\beta_{LF}}{\beta_{IF}}}$$

- Since the temperature change $\Delta T$ can be measured by $V_{AC}$ and the sum of the system and antenna temperatures by $V_{DC}$ the two ratios will be the same

$$\frac{\Delta T}{T_A + T_{sys}} = \sqrt{\frac{2\beta_{LF}}{\beta_{IF}}}$$

## Detection(2)

- Rewriting in terms of the temperature difference

$$\Delta T = (T_A + T_{sys})\sqrt{\frac{2\beta_{LF}}{\beta_{IF}}}$$

- If the lowpass filter is implemented as an integrator with a time constant $\tau$ (seconds) then the bandwidth $\beta_{LF}$ = 1/2$\tau$ and the formula becomes

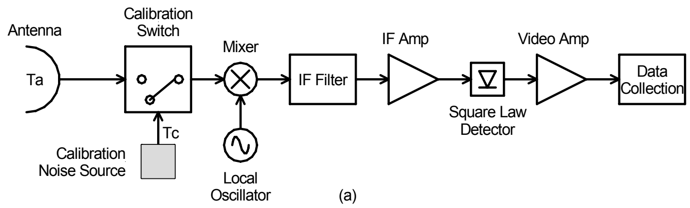
$$\Delta T = \frac{T_A + T_{sys}}{\sqrt{\beta_{IF}\tau}}$$

  - Note that $\beta_{IF}$ is not the 3dB bandwidth but the reception bandwidth which for a 2 pole RC filter $\beta_{IF}$ = 1.96$\beta_{3dB}$
  - The temperature difference calculated here is for a unity signal to noise ratio. Hence if good detection probability is required then a larger $\Delta T$ is required

## Characteristics of the Sensitivity Equation

- Measurable temperature difference $\Delta T$ decreases with:
  - Decreasing system temperature $T_{sys}$
  - Increasing IF bandwidth $\beta_{IF}$
  - Increasing integration time $\tau$
- The system temperature can be reduced by using low loss components, by placing a low noise amplifier at the antenna (not feasible at 94GHz) or by cooling the receiver
- The IF bandwidth can be widened, but it is seldom possible to obtain components with bandwidths in excess of 10% of the centre frequency
- Integration time can be traded off against the total observation time required when scanning an image

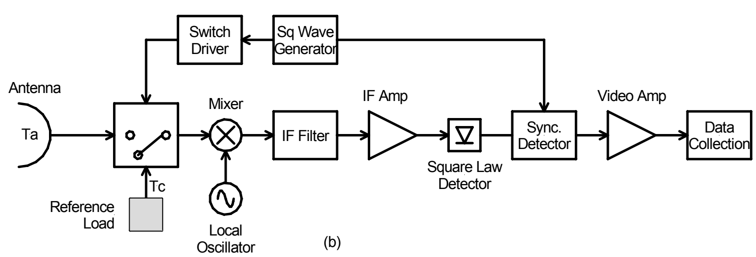
## Total Power Radiometer




- A square law detector cannot distinguish between an increase in the signal power caused by an increase in $T_A$ and an increase in the pre-detection gain $G$
- If the gain varies by $\Delta G$ around the average $G$, then the minimum detectable temperature change $\Delta T_{min}$ becomes

$$\Delta T_{\min} = (T_A + T_{sys})\left[\frac{1}{\beta_{IF}\tau} + \left(\frac{\Delta G}{G}\right)^2\right]^{1/2}$$

## Dicke Radiometer




- In the Dicke radiometer the receiver input spends 50% of the time observing the antenna temperature and the other 50% looking at a reference load
- The output of the square law detector is then synchronously detected to produce an output that is proportional to the difference between $T_c$ and $T_A$

$$\Delta T_{\min} = (T_A + 2T_{sys} + T_c)\left[\frac{1}{\beta_{IF}\tau} + \left(\frac{\Delta G}{G}\cdot\frac{T_A - T_c}{T_A + 2T_{sys} + T_c}\right)^2\right]^{1/2}$$

## Radiometer IF and Video Gain

- For a typical uncooled radiometer, the antenna temperature will be about $T_A \approx$ 300K and the system temperature $T_{sys} \approx$ 1500K
- For a bandwidth of 2GHz, the total received power is

$$P_N = k(T_A + T_{sys})\beta_{IF}$$

$$P_N = 4.97 \times 10^{-11} W$$

$$P_{NdB} = 10\log_{10} P_N + 30 = -73dBm$$

- The detector input should be about -10dBm so an IF gain of about 63dB is required in this case
- For a 200Ω load, this is about 10mV, so for a final output voltage of 1V a video gain of 40dB is required

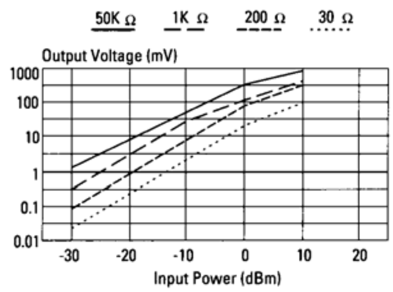
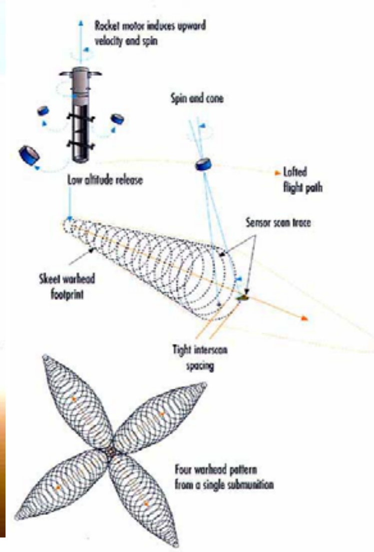



**Figure 1. Typical transfer characteristics.**

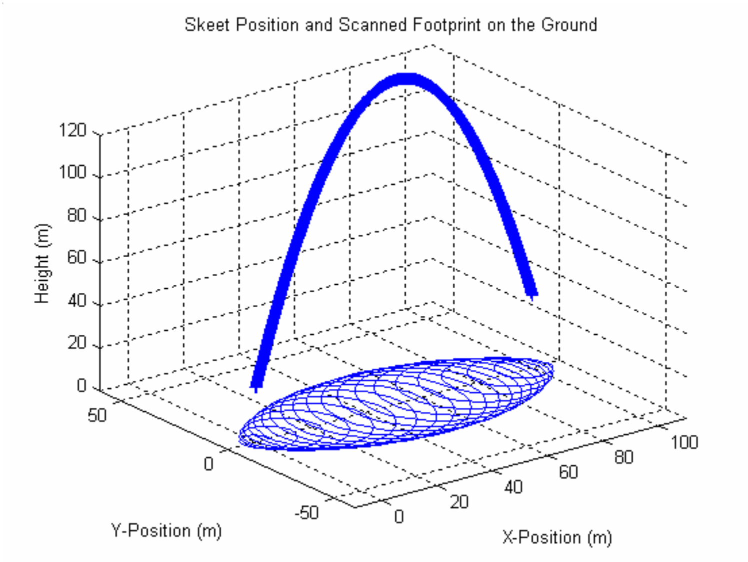
## Example: Anti Tank Skeet Design



## Skeet Characteristics

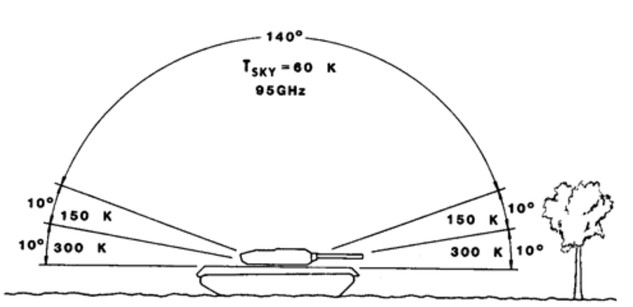
- Assume that the skeet motion is as follows
  - Launched from a height of 25m
  - Upward velocity 50m/s
  - Horizontal velocity 10m/s
  - Cone angle 10°
  - Spin rate 2rps
- Radiometric seeker characteristics
  - Aperture 50mm
  - Operational frequency 94GHz
  - IF bandwidth 2GHz

## The Search Pattern




## Environment and Tank Characteristics

- The tank temperature is 35°C (308K) as it has been driving
- The tank is made of rough steel with an emissivity $\varepsilon = 0.1$
- The surrounding terrain is 20°C and it is grass and soil with $\varepsilon = 0.92$
- The air is clear



## The Temperature of the Tank

- The average temperature of the tank over the full hemisphere is the sum of the various reflected temperatures scaled by their various areas and the tank reflectivity plus the emitted temperature scaled by the tank emissivity.
- By integrating over the surface of the sphere, it can be shown that the approximate area of the 10° sections are

  $A_1 = A_2 = 1.09$ steradians
- Therefore the area of the remaining 140° section will be

  $A_3 = 2\pi-(A_1+A_2) = 4.1$ steradians
- The reflected (scattered) temperature is

$$T_{SC} = \rho\left[\frac{A_1}{2\pi}T_1 + \frac{A_2}{2\pi}T_2 + \frac{A_3}{2\pi}T_3\right] = 0.9\left[\frac{1.09}{2\pi}300 + \frac{1.09}{2\pi}150 + \frac{4.10}{2\pi}60\right] = 105.5K$$

- And the radiated (brightness) temperature is

$$T_{BT} = \varepsilon T_T = 0.1 \times 308 = 30.8K$$

## Temperature of the Tank (2)

- The apparent temperature is the sum of the reflected and radiated temperatures modified by the atmospheric loss plus the upwelling temperature
- For a path length of 100m and clear air attenuation of 0.2dB/km the loss is small $L_A \approx 1$.
- The upwelling temperature which is related to the attenuation is very small $T_{up} \approx 1$K so is ignored

$$T_{AP} = T_{UP}(\theta,\phi) + \frac{1}{L_A}\left[T_{BT} + T_{SC}\right] \approx 30.8 + 105.5 = 136.3K$$

## Temperature of the background

- For a ground temperature of 20°C (293K) with an emissivity of 0.92, the apparent temperature of the ground is calculated in the same way as it is for the target.
- The average downwelling temperature is

$$T_{DN} = \left[\frac{A_1}{2\pi}T_1 + \frac{A_2}{2\pi}T_2 + \frac{A_3}{2\pi}T_3\right] = \left[\frac{1.09}{2\pi}300 + \frac{1.09}{2\pi}150 + \frac{4.10}{2\pi}60\right] = 117K$$

- Note in this case that the scattered temperature is very low because the reflectivity of the ground is low

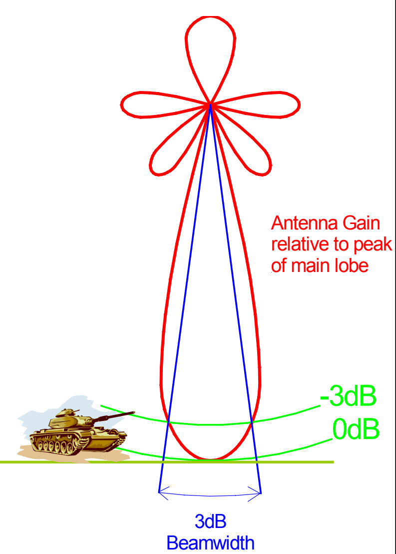
$$T_{APG} = \varepsilon_G T_G + \rho T_{GSC} = 0.92 \times 293 + 0.08 \times 117 = 279K$$

## Antenna Pattern

A reasonable approximation of the 3dB beamwidth of an antenna is given by the following formula

$$\theta_{3dB} = \frac{1.22\lambda}{D} \quad \text{radians}$$

Where $\lambda$ - Wavelength (m)
$D$ – Antenna diameter (m)



## Beamfill Effects

- For an antenna with a 3dB beamwidth of $\theta_{3dB}$ the footprint area on the ground is

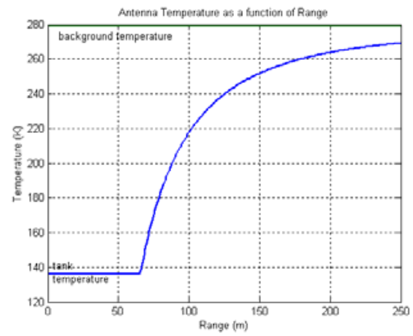
$$A_B = \frac{\pi}{4} d^2 = \frac{\pi}{4} (R\theta_{3dB})^2$$

- For $D$=50mm and $\lambda$=3.16mm this equates to

$A_B = 4.67 \times 10^{-3} R^2$ m$^2$

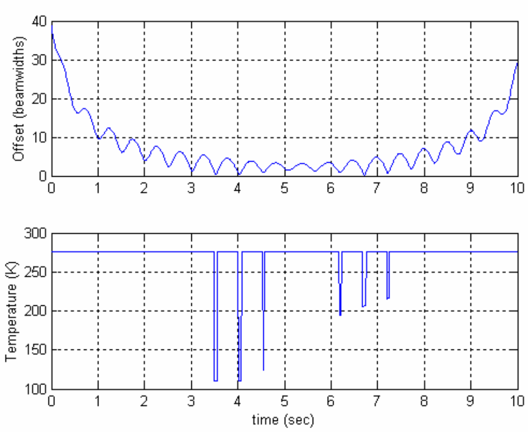
- The cross sectional area of a tank seen from above $A_T$ = 20m$^2$ so the scene temperature measured by the antenna is the sum of the brightness areas scaled by their relative areas

$$T_A = T_{APG} \frac{A_B - A_T}{A_B} + T_{APT} \frac{A_T}{A_B}$$




## Antenna Temperature

- As the antenna scans across the terrain, it will measure the background temperature of 279K, when it encounters the tank, the apparent temperature will dip
- The amount of dip will depend on the beamfill (which is proportional to range)



## Radiometer Implementation (Integration)

- The allowable integration time is limited to the dwell time of the beam on the target.
- For a cone angle of 10° (0.17rad), the circumference of the circle scanned on the ground is
  $circ = 2\pi(R\theta_{cone})$
- And the diameter of the footprint on the ground is
  $D_{foot} = R\frac{1.22\lambda}{D}$
- So, as the skeet spins at 2rps, the dwell time in seconds is

$$\tau = \frac{D_{foot}}{circ}\frac{1}{2} = \frac{1.22\lambda}{4\pi D\theta_{cone}} = \frac{1.22 \times 3.21}{4\pi \times 50 \times 0.17} = 37ms$$

## Receiver Noise Temperature

- We assume single sideband operation and that the mixer is placed at the focal point of the antenna to minimise waveguide losses.
  - $L$ = 0.2dB = 1.05 (feed loss from the antenna to the mixer)
  - $L_m$ = 6dB = 3.98 (mixer conversion loss)
  - $NF_{IF}$ = 1.5dB = 1.41 (low noise amplifier noise figure)

$$NF_{SSB} = L.L_M\, NF_{IF} = 1.05 \times 3.98 \times 1.41 = 5.88$$

$$T_{SYS} = T_O(NF_{SSB} - 1) = 290(5.88 - 1) = 1415K$$

## Noise Figure of a Single Sideband Receiver

- The total noise figure of a cascaded receiver chain is as follows

$$NF = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2}$$

- For this case $L=NF_1$ $L_M=NF_2$ $1/L=G_1$ $1/L_m=G_2$ and $NF_{IF}=NF_3$

$$NF_{SSB} = L + L(L_M - 1) + (NF_{IF} - 1)L.L_M = L.L_M.NF_{IF}$$

## Minimum Detectable Temperature Difference

- For a total power radiometer, we assume that the gain is completely stable $\Delta G = 0$, so we can use the following formula

$$\Delta T_{\min} = \frac{T_A + T_{sys}}{\sqrt{\beta_{IF}\tau}} = \frac{279 + 1415}{\sqrt{2\times10^9 \times 37\times10^{-3}}} = 0.2K$$

Where $T_A$ – Background Temperature (279K)
$T_{sys}$ – Receiver System Temperature (1415K)
$\beta_{IF}$ – Receiver Bandwidth (2GHz)
$\tau$ - Integration time (37ms)

- We have shown in an earlier lecture that for a detection probability of 90% with a false alarm probability of $10^{-6}$ a signal to noise ratio of 13dB is required
- This equates to a temp. change of $0.2\text{x}10^{13/10}$ = 4K which will occur at a range of 420m

## Amplifier Gains

- The total power at the antenna including the scene temperature and the system temperature is

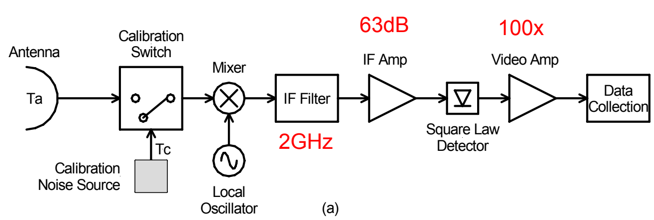
$$P = 30 + 10\log_{10} k(T_A + T_{sys})\beta = -73dBm$$

- For the square law detector shown earlier, an input power of about -10dBm is required, so the IF gain $G_{IF}$ required is

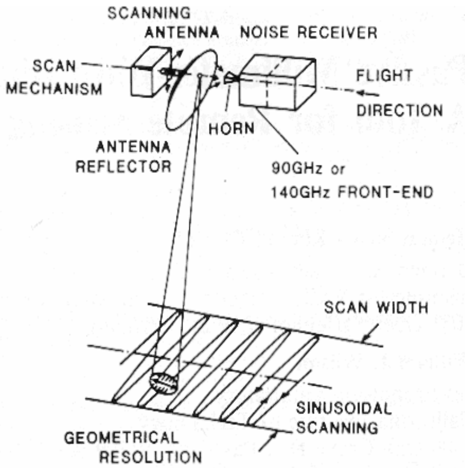
$$G_{IF} = -10 + 73 = 63dB$$

- This will result in an output voltage of 10mV.
- For detection purposes a DC voltage of about 1V is required, so a voltage gain of 100 will be needed

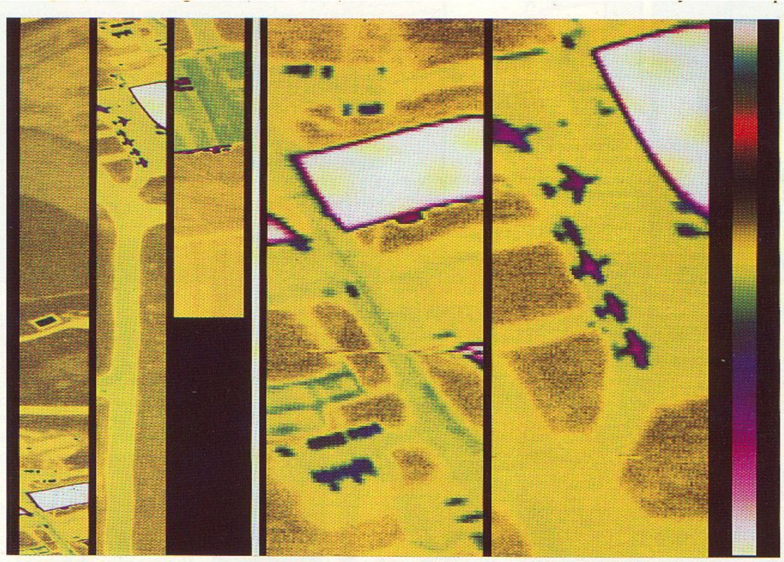
## Radiometer



## Airborne Push-Broom Radiometer

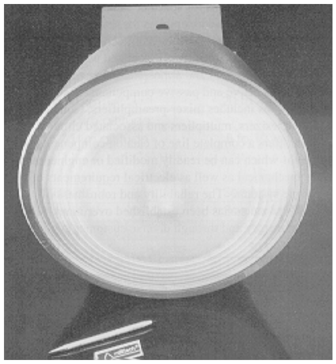



- Provide an imaging capability through cloud, fog, smoke and dust that are opaque at IR wavelengths
- Do not generally provide the same resolution as that available from IR sensors




Flight with the 140 GHz imaging sensor over an airport. a) shows the successive parts of the total flight; b) and c) shows enlargements of characteristic patterns. Altitude = 125 m; lenght of the flight path = 1730 m, scanwidth = 60 m.

## Radiometric Cameras



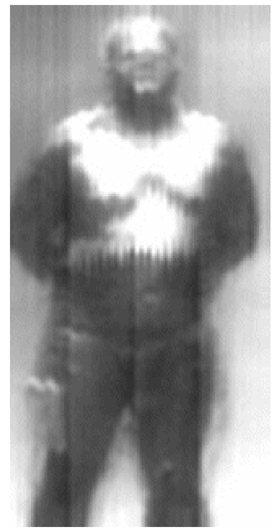
Millitech Unit

Langley Research Unit

## Radiometric Cameras For Security

- Radiometric images penetrate clothes
- See plastic ceramic or metal weapons
- See explosives

## More Radiometric Cameras For  Security



## Radiometric Imagers For Outdoor Guidance In Bad Weather

Visible & 94GHz Radiometric Images of the Severn Valley

## Radiometric Images – Super Resolution

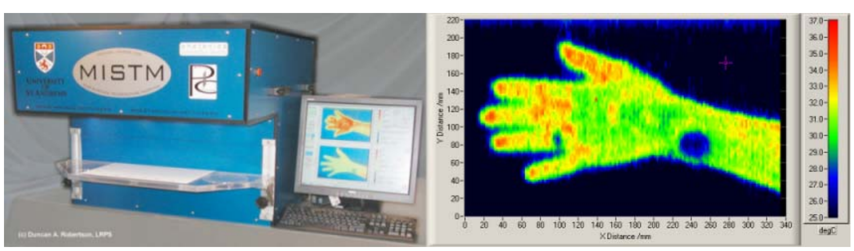
Videoimage.

95 GHz passive millimeter wave image.

Visible & 94GHz Radiometric Images of the Car Park

## Medical Imaging

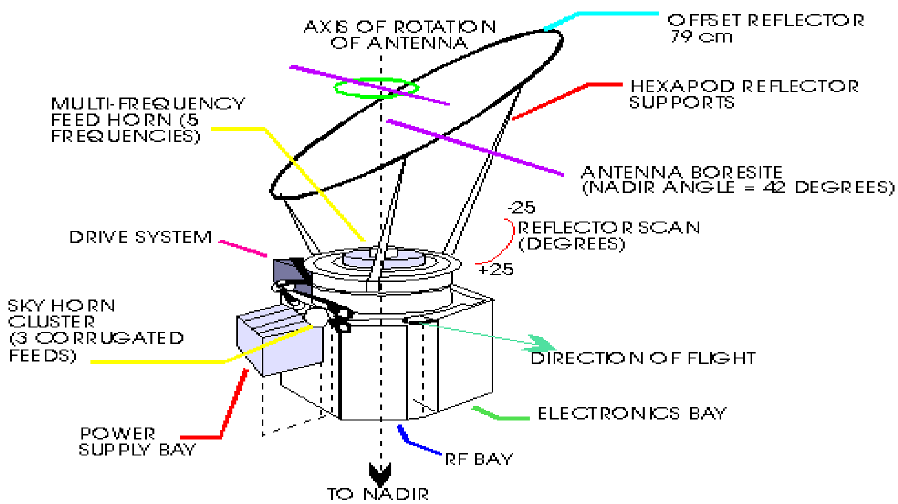
- St Andrews University imaging radiometer for medical research
- Millimetre wave radiation penetrates the top mm
  - Useful in examining skin lesions, cancers etc.
  - Can see through clothing to determine physical damage of trauma patients



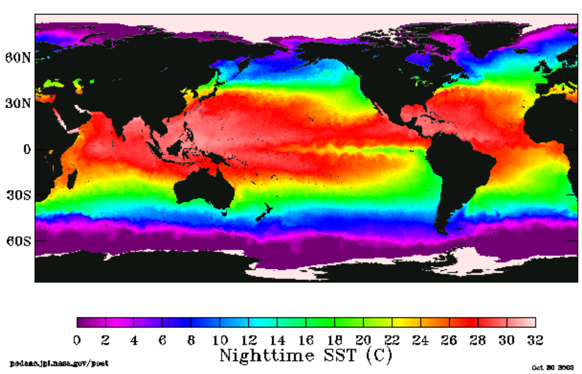

Radiometer

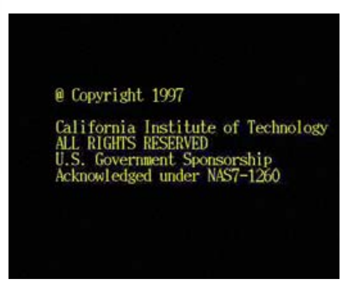
Radiometric Image

## Space Based Microwave Radiometer




## Space Based Microwave Radiometric Imaging



## Millimetre Wave Radio Astronomy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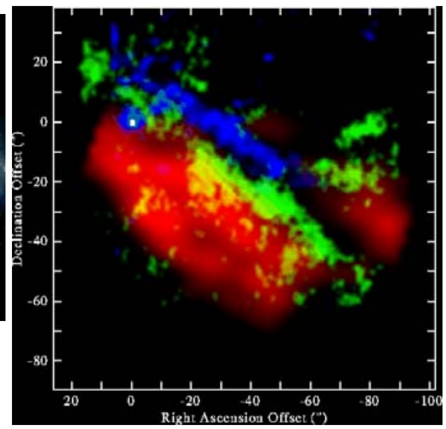
- Millimetre wave radio telescopes are used to detect many species of molecules in interstellar clouds by their absorption and emission spectra.
- The Mopra radio telescope in NSW has been upgraded by the CSIRO and University of NSW to have a solid surface accurate to 0.25mm over its full 22m diameter.
- The primary task of single telescopes is to survey large regions of the sky looking for objects suitable for scrutiny by the large arrays.
- The BIMA array is a 10 antenna aperture synthesis
- Each telescope is 6.1m in diameter with a surface that is accurate to 30um rms.
- The antennas can be placed anywhere on a T with separations between 7m and 2km.

## Visible And Millimetre Wave Images Of Orion




## BIMA Investigation Of Molecules Around Stars